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Chapman–Enskog solutions to arbitrary order in Sonine polynomials II: Viscosity in a binary, rigid-sphere, gas mixture

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ABSTRACT

The Chapman–Enskog solutions of the Boltzmann equations provide a basis for the computation of important transport coefficients for both simple gases and gas mixtures. These coefficients include the viscosity, the thermal conductivity, and the diffusion coefficient. In a preceding paper (I), for simple, rigid-sphere gases (i.e. single-component, monatomic gases) we have shown that the use of higher-order Sonine polynomial expansions enables one to obtain results of arbitrary precision that are error free. It is our purpose in this paper to report the results of our investigation of relatively high-order, standard, Sonine polynomial expansions for the viscosity-related Chapman–Enskog solutions for binary gas mixtures of rigid-sphere molecules. We note that in this work we have retained the full dependence of the solution on the molecular masses, the molecular sizes, the mole fractions, and the intermolecular potential model via the omega integrals. For rigid-sphere gases, all of the relevant omega integrals needed for these solutions are analytically evaluated and, thus, results to any desired precision can be obtained. The values of viscosity obtained using Sonine polynomial expansions for the Chapman–Enskog solutions converge monotonically from below and, therefore, the exact viscosity solution to a given degree of convergence can be determined with certainty by expanding to sufficiently high an order. We have used *Mathematica*® for its versatility in permitting both symbolic and high precision computations. Our results also establish confidence in the results reported recently by other authors who used direct numerical techniques to solve the relevant Chapman–Enskog equations. While in all of the direct numerical methods more-or-less full calculations need to be carried out with each variation in molecular parameters, our work utilizes explicit, general expressions for the necessary matrix elements that retain the complete parametric dependence of the problem and, thus, only a matrix inversion at the final step is needed as a parameter is varied. This work also indicates how similar results may be obtained for more realistic intermolecular potential models and how other gas-mixture problems may also be addressed with some additional effort.

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1. Introduction

The Chapman–Enskog solutions of the Boltzmann equations provide a basis for the computation of important transport coefficients for both simple gases and gas mixtures [1–15]. The use of Sonine polynomial expansions for the Chapman–Enskog solutions was first suggested by Burnett [16] and has become the general method for obtaining the transport coefficients due to the relatively rapid convergence of this series [1–8,16,17]. While it has been found that relatively, low-order expansions (of order 4) can provide reasonable accuracy in computations of the transport coefficients (to about 1 part in 1000), the adequacy of the low-order expansions for computation of the slip and jump coefficients associated with gas-surface interfaces still needs to be explored. Also of importance is the fact that such low-order expansions do not provide good convergence (in velocity space) for the actual Chapman–Enskog solutions even though the transport coefficients derived from these solutions appear to be reasonable. Thus, it is of some interest to explore Sonine polynomial expansions to higher orders. In a preceding paper [17], we have shown for simple, rigid-sphere gases (i.e. single-component, monatomic gases) that, indeed, the use of higher-order Sonine polynomial expansions enables one to obtain results of arbitrary precision that are error free. It is our purpose in this paper to report the results of our investigation of relatively high-order, standard, Sonine polynomial expansions for the viscosity-related Chapman–Enskog solutions for binary gas mixtures of rigid-sphere molecules. In the following sections we describe the basic theory, the theory relating to viscosity, the solution technique in terms of the Sonine polynomials, the bracket integrals, details related to the specific case of rigid-spheres molecules, and our results.

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A part of our motivation with respect to this work has been some of the recently reported results on direct numerical solutions of the linearized Boltzmann equations for rigid-sphere, gas mixtures. In particular, results for the transport coefficients and the Chapman–Enskog solutions have been reported both by Takata et al. [18] and by Garcia and Siewert [19]. Our work provides a benchmark for assessing the precision of the numerical results reported by these authors and, indeed, we report some such comparisons that we have made. Our work does have an important distinguishing feature in that, for rigid-sphere gas mixtures, we require no numerical integrations and thus, in principle, results of arbitrary precision can be obtained for any given order of the Sonine polynomial expansions. We note that the computational resources available to us at the present time have realistically permitted expansions only to order 60 given the manner in which we have implemented this technique but that, even here, it has been possible to obtain convergence of 9 or more significant digits in the normalized gas mixture viscosities (depending upon the specific mass ratios, size ratios, and mole fractions considered) and it is certain that further improvements in the implementation of the technique or the availability of better computational resources will allow even higher-order expansions and greater convergence of the results. Further, we note that in this work we have retained the full dependence of the solutions on the molecular masses, the molecular sizes, the mole fractions, and the intermolecular potential model via the omega integrals, and we have obtained explicit (symbolic) expressions for the necessary matrix elements (the bracket integrals) used in evaluating the coefficients in the Sonine polynomial expansions for the coupled Chapman–Enskog equations. These generalized matrix elements, once determined, need not be determined again. For rigid spheres (or for any other potential model of interest that can be represented via the omega integrals), we can then determine in a straightforward manner a set of matrix elements that are specific to the potential model being used and store them. These specific matrix elements require only the input of the appropriately computed omega integrals which, for rigid spheres, are known exactly such that no numerical integrations are needed. In this fashion, our method requires only a matrix inversion at the final step. This is important, as all that is needed for finding both the viscosity and the Chapman–Enskog viscosity solutions for arbitrary, binary, rigid-sphere gas mixtures is precomputed in a general form. Thus, we are able to study parametric dependencies and convergence of our results in an economical and systematic way as, once the matrix elements up to the highest order are computed and stored, we can process results to any order up to this highest order without any new computations of matrix elements being required. Further, since our values for the viscosity converge monotonically with increasing order, since we can use arbitrarily high numerical precision as needed in *Mathematica*® for the final matrix inversion step, and since we can easily compare results for a given order with the results for immediately preceding orders, we can be confident in our results and the degree of convergence obtained.

2. The basic theory

Following the work and notations of Chapman and Cowling [1], we offer below an abbreviated version of the relevant theory. For an arbitrary, rarefied, gas mixture, one begins with the Boltzmann equations describing the molecular distribution functions of the constituent gases:

$$\left(\frac{\partial}{\partial t}+\mathbf{c}_i\cdot\nabla_r+\mathbf{F}_i\cdot\nabla_{c_i}\right)f_i(\mathbf{r},\mathbf{c}_i,t)$$
$$=\sum_j\iiint(f_i'f_j'-f_if_j)gb\,\mathrm{d}b\,\mathrm{d}\varepsilon\,\mathrm{d}\mathbf{c}_j$$
$$=\sum_j J(f_if_j),\tag{1}$$

in which the left-hand side (LHS) is known as the streaming term of the equation which contains the differential streaming operator in the brackets, the right-hand side (RHS) is a sum over what are known as the collision integrals in which $J(f_if_j)$ is called the collision operator, $f_i(\mathbf{r},\mathbf{c}_i,t)$ is the molecular distribution function of the ith constituent, g is the magnitude of the pre-collision relative velocity, $\mathbf{g}=\mathbf{c}_j-\mathbf{c}_i$, b is the 'impact parameter' associated with the binary scattering events, ε is an angle corresponding to the azimuthal orientation of the scattering plane, and $\mathbf{c}$ is the molecular velocity. A prime ($'$) indicates a function of a post-collision velocity while the corresponding lack of a prime indicates a pre-collision velocity dependence, e.g. $f_i=f_i(\mathbf{r},\mathbf{c}_i,t)$ while $f_i'=f_i(\mathbf{r},\mathbf{c}_i',t)$. In the summation over the different constituents, scattering between like constituents (i.e. when $i=j$) is treated in the same way as scattering between unlike constituents with the various pre- and post-collision velocities retained as separate variables for purposes of integration. In this circumstance, for clarity, it is common practice to drop the i subscript inside the collision integral in order to facilitate the necessary discrimination between the velocities (i.e. $\mathbf{c}_i\to\mathbf{c}$ and $f_i\to f$). Of course it follows from this that, if one is dealing with a simple gas having only one constituent, one obtains from this process the single Boltzmann equation describing the gas in which $j=1$ and no subscript is necessary on the LHS:

$$\left(\frac{\partial}{\partial t}+\mathbf{c}\cdot\nabla_r+\mathbf{F}\cdot\nabla_c\right)f(\mathbf{r},\mathbf{c},t)$$
$$=\iiint(f'f_1'-ff_1)gb\,\mathrm{d}b\,\mathrm{d}\varepsilon\,\mathrm{d}\mathbf{c}_1.\tag{2}$$

Equivalent expressions for the above equations are often encountered in which $b\,\mathrm{d}b\,\mathrm{d}\varepsilon$ is expressed as $\alpha_{ij}(g,\chi)\,\mathrm{d}\mathbf{e}'$ or $\sigma_{ij}(g,\chi)\,\mathrm{d}\mathbf{\Omega}$ where χ is the scattering angle (the angle between $\mathbf{g}$ and $\mathbf{g}'$) and $\alpha_{ij}(g,\chi)=\sigma_{ij}(g,\chi)$ is known as the differential collision cross-section which describes the probability per unit time per unit volume that two molecules colliding with velocities, $\mathbf{c}_i$ in $\mathrm{d}\mathbf{c}_i$ and $\mathbf{c}_j$ in $\mathrm{d}\mathbf{c}_j$, will have a relative velocity after collision, $\mathbf{g}'=\mathbf{c}_j'-\mathbf{c}_i'$, that lies within the solid angle, $\mathrm{d}\mathbf{e}'=\mathrm{d}\mathbf{\Omega}=\sin(\chi)\,\mathrm{d}\chi\,\mathrm{d}\varepsilon$.

For the specific case of a binary gas mixture, one expresses the distribution functions f_1 and f_2 in the form:

$$f_1=f_1^{(0)}+f_1^{(1)}+f_1^{(2)}+\cdots,\tag{3}$$
$$f_2=f_2^{(0)}+f_2^{(1)}+f_2^{(2)}+\cdots,\tag{4}$$

where the lowest-order approximations are chosen to be:

$$f_1^{(0)}=n_1\left(\frac{m_1}{2\pi kT}\right)^{3/2}\exp\left(-\frac{m_1}{2kT}(\mathbf{c}_1-\mathbf{c}_0)^2\right),\tag{5}$$
$$f_2^{(0)}=n_2\left(\frac{m_2}{2\pi kT}\right)^{3/2}\exp\left(-\frac{m_2}{2kT}(\mathbf{c}_2-\mathbf{c}_0)^2\right),\tag{6}$$

in which m_1 and m_2 are the molecular masses of the constituent gases, k is Boltzmann's constant, and n_1, n_2, $\mathbf{c}_0$ and T are, in general, arbitrary functions of $\mathbf{r}$ and t. Note that in choosing the lowest-order approximations to be of this form (which correspond to Maxwellian distributions), one has effectively equated the functions n_1 and n_2 to the number densities of the two gases in the mixture, T to the temperature of the mixture, and $\mathbf{c}_0$ to the mass-velocity of the mixture where, $\mathbf{c}_0=M_1x_1\mathbf{c}_1+M_2x_2\mathbf{c}_2$ in which, $M_i=m_i/m_0$, $m_0=m_1+m_2$, $x_i\equiv n_i/n$ denote the proportion by number of the constituent gases in the mixture (the mole fractions), and $n=n_1+n_2$ is the total molecular number density of the binary mixture.

If one now limits further consideration only to the second approximation (up to $f^{(1)}$), that is equivalent to assuming that the distribution functions for each of the constituents can be expressed as small linear perturbations from equilibrium states specified by the Maxwellian distributions of Eqs. (5) and (6), i.e. $f_i = f_i^{(0)}(1+\Phi_i^{(1)})$. Thus, $f_1^{(1)}$ and $f_2^{(1)}$ are written in the form:

$$f_1^{(1)} = f_1^{(0)}\Phi_1^{(1)}, \tag{7}$$

$$f_2^{(1)} = f_2^{(0)}\Phi_2^{(1)}, \tag{8}$$

where the perturbations, $\Phi_1^{(1)}$ and $\Phi_2^{(1)}$, satisfy the time derivative expressions given by:

$$\mathcal{D}_1^{(1)} = -n_1^2 I_1\big(\Phi_1^{(1)}\big) - n_1 n_2 I_{12}\big(\Phi_1^{(1)} + \Phi_2^{(1)}\big), \tag{9}$$

$$\mathcal{D}_2^{(1)} = -n_2^2 I_2\big(\Phi_2^{(1)}\big) - n_1 n_2 I_{21}\big(\Phi_1^{(1)} + \Phi_2^{(1)}\big), \tag{10}$$

in which:

$$\mathcal{D}_i^{(r)} = \frac{\partial_{r-1} f_i^{(0)}}{\partial t} + \frac{\partial_{r-2} f_i^{(1)}}{\partial t} + \cdots + \frac{\partial_0 f_i^{(r-1)}}{\partial t} + \left(\mathbf{c}_i \cdot \frac{\partial}{\partial \mathbf{r}} + \mathbf{F}_i \cdot \frac{\partial}{\partial \mathbf{c}_i}\right) f_i^{(r-1)}, \tag{11}$$

and:

$$n_i^2 I_i(F) = \iint f_i^{(0)} f^{(0)} (F_i + F - F_i' - F') g\alpha_i \, \mathrm{d}\mathbf{e}' \, \mathrm{d}\mathbf{c}, \tag{12}$$

$$n_i n_j I_{ij}(K) = \iint f_i^{(0)} f_j^{(0)} (K - K') g\alpha_{ij} \, \mathrm{d}\mathbf{e}' \, \mathrm{d}\mathbf{c}_j, \tag{13}$$

are linear functions of their arguments such that $I(\phi+\psi) = I(\phi) + I(\psi)$ and $I(a\phi) = aI(\phi)$ regardless of subscripts (where a is any arbitrary constant). The LHSs of Eqs. (9) and (10) can be expressed in the form:

$$\mathcal{D}_1^{(1)} = f_1^{(0)}\left\{\left(\mathscr{C}_1^2 - \frac{5}{2}\right)\mathbf{C}_1 \cdot \nabla \ln(T) + x_1^{-1}\mathbf{d}_{12}\cdot \mathbf{C}_1 + 2\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1} : \nabla \mathbf{c}_0\right\}, \tag{14}$$

$$\mathcal{D}_2^{(1)} = f_2^{(0)}\left\{\left(\mathscr{C}_2^2 - \frac{5}{2}\right)\mathbf{C}_2 \cdot \nabla \ln(T) + x_2^{-1}\mathbf{d}_{21}\cdot \mathbf{C}_2 + 2\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2} : \nabla \mathbf{c}_0\right\}, \tag{15}$$

in which $\boldsymbol{\mathscr{C}}_i \equiv (m_i/2kT)^{1/2}\mathbf{C}_i$, $\mathbf{C}_i \equiv \mathbf{c}_i - \mathbf{c}_0$, and the bold sans serif notation, w, denotes a dyadic tensor, $\mathsf{w} = \mathbf{ab}$, constructed from the components of the vectors, $\mathbf{a}$ and $\mathbf{b}$. Note that $\mathbf{d}_{12}$ is given as either of the two forms noted below:

$$\mathbf{d}_{12} = \frac{\rho_1\rho_2}{\rho p}\left\{\mathbf{F}_2 - \frac{1}{\rho_2}\nabla p_2 - \left(\mathbf{F}_1 - \frac{1}{\rho_1}\nabla p_1\right)\right\}, \tag{16}$$

$$\mathbf{d}_{12} = \nabla x_1 + \frac{n_1 n_2(m_2 - m_1)}{n\rho}\nabla \ln(p) - \frac{\rho_1\rho_2}{\rho p}(\mathbf{F}_1 - \mathbf{F}_2), \tag{17}$$

where ρ_1 and ρ_2 are the mass densities of the constituent gases, $\rho = \rho_1 + \rho_2$ is the total mass density of the mixture, p_1 and p_2 are the partial pressures of the constituent gases, and $p = p_1 + p_2$ is the total pressure of the mixture. Since $\nabla x_2 = -\nabla x_1$, either Eq. (16) or (17) can be used to show that $\mathbf{d}_{21} = -\mathbf{d}_{12}$. The functions $\Phi_1^{(1)}$ and $\Phi_2^{(1)}$ can then be expressed as:

$$\Phi_1^{(1)} = -\mathbf{A}_1 \cdot \frac{\partial \ln(T)}{\partial \mathbf{r}} - \mathbf{D}_1 \cdot \mathbf{d}_{12} - 2\mathsf{B}_1 : \frac{\partial}{\partial \mathbf{r}}\mathbf{c}_0, \tag{18}$$

$$\Phi_2^{(1)} = -\mathbf{A}_2 \cdot \frac{\partial \ln(T)}{\partial \mathbf{r}} - \mathbf{D}_2 \cdot \mathbf{d}_{12} - 2\mathsf{B}_2 : \frac{\partial}{\partial \mathbf{r}}\mathbf{c}_0, \tag{19}$$

where the functions $\mathbf{A}$ and $\mathbf{D}$ are vectors and the functions B are non-divergent tensors, such that:

$$\mathbf{A} = \mathbf{C}A(C), \quad \mathbf{D} = \mathbf{C}D(C), \quad \mathsf{B} = \overset{\circ}{\mathbf{CC}}B(C), \tag{20}$$

with the appropriate subscript 1 or 2 implied throughout each expression, and where $\mathbf{A}$, $\mathbf{D}$, and B, must satisfy the following pairs of equations, respectively:

$$f_1^{(0)}\left(\mathscr{C}_1^2 - \frac{5}{2}\right)\mathbf{C}_1 = n_1^2 I_1(\mathbf{A}_1) + n_1 n_2 I_{12}(\mathbf{A}_1 + \mathbf{A}_2), \tag{21}$$

$$f_2^{(0)}\left(\mathscr{C}_2^2 - \frac{5}{2}\right)\mathbf{C}_2 = n_2^2 I_2(\mathbf{A}_2) + n_1 n_2 I_{21}(\mathbf{A}_1 + \mathbf{A}_2), \tag{22}$$

$$x_1^{-1} f_1^{(0)}\mathbf{C}_1 = n_1^2 I_1(\mathbf{D}_1) + n_1 n_2 I_{12}(\mathbf{D}_1 + \mathbf{D}_2), \tag{23}$$

$$-x_2^{-1} f_2^{(0)}\mathbf{C}_2 = n_2^2 I_2(\mathbf{D}_2) + n_1 n_2 I_{21}(\mathbf{D}_1 + \mathbf{D}_2), \tag{24}$$

$$f_1^{(0)}\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1} = n_1^2 I_1(\mathsf{B}_1) + n_1 n_2 I_{12}(\mathsf{B}_1 + \mathsf{B}_2), \tag{25}$$

$$f_2^{(0)}\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2} = n_2^2 I_2(\mathsf{B}_2) + n_1 n_2 I_{21}(\mathsf{B}_1 + \mathsf{B}_2). \tag{26}$$

Note, that the forms for the distribution functions $\Phi_1^{(1)}$ and $\Phi_2^{(1)}$ have been chosen such that $\mathbf{A}$ and $\mathbf{D}$ must also satisfy the relationships:

$$\int f_1^{(0)} m_1 \mathbf{C}_1 \cdot \mathbf{A}_1 \, \mathrm{d}\mathbf{c}_1 + \int f_2^{(0)} m_2 \mathbf{C}_2 \cdot \mathbf{A}_2 \, \mathrm{d}\mathbf{c}_2 = 0, \tag{27}$$

$$\int f_1^{(0)} m_1 \mathbf{C}_1 \cdot \mathbf{D}_1 \, \mathrm{d}\mathbf{c}_1 + \int f_2^{(0)} m_2 \mathbf{C}_2 \cdot \mathbf{D}_2 \, \mathrm{d}\mathbf{c}_2 = 0, \tag{28}$$

such that the second-order Chapman–Enskog approximations yield:

$$f_1^{(1)} = f_1^{(0)}\big\{1 - A_1(C_1)\mathbf{C}_1 \cdot \nabla \ln(T) - D_1(C_1)\mathbf{C}_1 \cdot \mathbf{d}_{12} - 2B_1(C_1)\,\overset{\circ}{\mathbf{C}_1\mathbf{C}_1} : \nabla \mathbf{c}_0\big\}, \tag{29}$$

and:

$$f_2^{(1)} = f_2^{(0)}\big\{1 - A_2(C_2)\mathbf{C}_2 \cdot \nabla \ln(T) - D_2(C_2)\mathbf{C}_2 \cdot \mathbf{d}_{12} - 2B_2(C_2)\,\overset{\circ}{\mathbf{C}_2\mathbf{C}_2} : \nabla \mathbf{c}_0\big\}. \tag{30}$$

Eqs. (29) and (30) then allow one to verify that the mean kinetic energies of the peculiar motions of the molecules of each constituent gas are the same up to this level of approximation.

From Eqs. (21)–(26), one may then construct the following general expressions:

$$n^2\{\mathbf{A}, \mathbf{a}\} = \int f_1^{(0)}\left(\mathscr{C}_1^2 - \frac{5}{2}\right)\mathbf{C}_1 \cdot \mathbf{a}_1 \, \mathrm{d}\mathbf{c}_1 + \int f_2^{(0)}\left(\mathscr{C}_2^2 - \frac{5}{2}\right)\mathbf{C}_2 \cdot \mathbf{a}_2 \, \mathrm{d}\mathbf{c}_2, \tag{31}$$

$$n^2\{\mathbf{D}, \mathbf{a}\} = x_1^{-1}\int f_1^{(0)}\mathbf{C}_1 \cdot \mathbf{a}_1 \, \mathrm{d}\mathbf{c}_1 - x_2^{-1}\int f_2^{(0)}\mathbf{C}_2 \cdot \mathbf{a}_2 \, \mathrm{d}\mathbf{c}_2, \tag{32}$$

$$n^2\{\mathsf{B}, \mathsf{b}\} = \int f_1^{(0)}\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1} : \mathsf{b}_1 \, \mathrm{d}\mathbf{c}_1 + \int f_2^{(0)}\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2} : \mathsf{b}_2 \, \mathrm{d}\mathbf{c}_2, \tag{33}$$

where $\mathbf{a}$ is any vector-function defined in both velocity domains, b is any tensor function defined in both velocity domains, and $\{F, G\}$ are known as the bracket integrals which are defined as:

$$n^2\{F, G\} \equiv n_1^2[F, G]_1 + n_1 n_2[F_1 + F_2, G_1 + G_2]_{12} + n_2^2[F, G]_2, \tag{34}$$

where:

$$[F, G]_1 \equiv \int G_1 I_1(F)\, \mathrm{d}\mathbf{c}_1, \tag{35}$$

$$[F, G]_2 \equiv \int G_2 I_2(F)\, \mathrm{d}\mathbf{c}_2, \tag{36}$$

and:

$$[F_1 + G_2, H_1 + K_2]_{12} \equiv \int F_1 I_{12}(H_1 + K_2)\, \mathrm{d}\mathbf{c}_1 + \int G_2 I_{21}(H_1 + K_2)\, \mathrm{d}\mathbf{c}_2. \tag{37}$$

Here, due to symmetry and linearity, one has that $[F,G]_1 = [G,F]_1$, $[F,G]_2 = [G,F]_2$, and $[F_1 + G_2, H_1 + K_2]_{12} = [H_1 + K_2, F_1 + G_2]_{12}$ such that $\{F,G\} = \{G,F\}$, $\{F, G + H\} = \{F,G\} + \{F,H\}$, and $\{F, aG\} = a\{F,G\}$ (where a is any arbitrary constant).

3. The theory for viscosity

Focusing solely on the viscosity solution, one notes that the lowest-order approximation to the pressure system for a binary gas mixture reduces simply to the equilibrium hydrostatic pressure of the mixture. For the second-order approximation, one then adds to this the pressure system $\mathbf{p}^{(1)}$ given by:

$$\mathbf{p}^{(1)} \equiv n_1 m_1 (\overline{\mathbf{C}_1\mathbf{C}_1})^{(1)} + n_2 m_2 (\overline{\mathbf{C}_2\mathbf{C}_2})^{(1)}, \tag{38}$$

which represents the deviation of the pressure system from the hydrostatic value. Now, in terms of the second approximation distribution functions of Eqs. (29) and (30):

$$\begin{aligned}\mathbf{p}^{(1)} &= -2m_1 \int f_1^{(0)} \mathbf{C}_1\mathbf{C}_1 (\mathbf{B}_1 : \nabla \mathbf{c}_0)\, d\mathbf{c}_1 \\ &\quad - 2m_2 \int f_2^{(0)} \mathbf{C}_2\mathbf{C}_2 (\mathbf{B}_2 : \nabla \mathbf{c}_0)\, d\mathbf{c}_2 \\ &= -\frac{2}{5}\left\{ m_1 \int f_1^{(0)}\, \overset{\circ}{\mathbf{C}_1\mathbf{C}_1} : \mathbf{B}_1\, d\mathbf{c}_1 + m_2 \int f_2^{(0)}\, \overset{\circ}{\mathbf{C}_2\mathbf{C}_2} : \mathbf{B}_2\, d\mathbf{c}_2 \right\} \overset{\circ}{\mathbf{e}} \\ &= -\frac{4}{5} k n^2 T \{\mathbf{B}, \mathbf{B}\} \overset{\circ}{\mathbf{e}},\end{aligned} \tag{39}$$

where $\mathbf{e} \equiv \overline{\overline{\nabla \mathbf{c}_0}}$ is the rate-of-strain tensor and $\overset{\circ}{\mathbf{e}} \equiv \overset{\circ}{\overline{\overline{\nabla \mathbf{c}_0}}}$ is the rate-of-shear tensor. If one then writes the viscosity as:

$$\mu \equiv \frac{2}{5} k n^2 T \{\mathbf{B}, \mathbf{B}\}, \tag{40}$$

Eq. (39) becomes:

$$\mathbf{p}^{(1)} = -2\mu \overset{\circ}{\mathbf{e}} \equiv -2\mu\, \overset{\circ}{\overline{\overline{\frac{\partial}{\partial \mathbf{r}}}}} \mathbf{c}_0, \tag{41}$$

which is the expression for the viscous stress system in any medium.

4. Solution in terms of Sonine polynomials

For the Chapman–Enskog functions of viscosity of a binary mixture, $\mathbf{B}_1$ and $\mathbf{B}_2$, it is assumed that both $\mathbf{B}_1$ and $\mathbf{B}_2$ can be expressed in series form as follows:

$$\mathbf{B}_1 = \sum_{\substack{p=-\infty \\ p\neq 0}}^{+\infty} b_p \mathbf{b}_1^{(p)}, \qquad \mathbf{B}_2 = \sum_{\substack{p=-\infty \\ p\neq 0}}^{+\infty} b_p \mathbf{b}_2^{(p)}, \tag{42}$$

where the same expansion coefficients, b_p, are used in both series, the value $p = 0$ is explicitly not included in either series (Note: in Chapman and Cowling [1], summations that explicitly omit the 0th term are denoted with primes on the summation symbol), and the functions, $\mathbf{b}^{(p)}$, in the two series are defined in the two velocity domains by the equations:

$$\begin{aligned} &\mathbf{b}_1^{(p)} \equiv 0, && \mathbf{b}_2^{(-p)} \equiv 0, && (p < 0), \\ &\mathbf{b}_1^{(p)} \equiv S_{5/2}^{(p-1)}\big(\mathscr{C}_1^2\big)\, \overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, && \mathbf{b}_2^{(-p)} \equiv S_{5/2}^{(p-1)}\big(\mathscr{C}_2^2\big)\, \overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}, && (p > 0), \end{aligned} \tag{43}$$

where:

$$\begin{aligned} S_m^{(n)}(x) &= \sum_{p=0}^{n} \frac{(m+n)_{n-p}}{(p)!(n-p)!}(-x)^p \\ &= \sum_{p=0}^{n} \frac{(m+n)!}{(p)!(n-p)!(m+p)!}(-x)^p, \end{aligned} \tag{44}$$

(with $S_m^{(0)}(x) = 1$ and $S_m^{(1)}(x) = m + 1 - x$) are numerical multiples (un-normalized) of the Sonine polynomials originally used in the Kinetic Theory of Gases by Burnett [16]. As it is used in Chapman and Cowling (and in the present work), we note the following orthogonality property of the Sonine polynomials:

$$\int_0^\infty \exp(-x) S_m^{(p)}(x) S_m^{(q)}(x) x^m\, dx = \big\{\Gamma(m+p+1)/p!\big\}\delta_{p,q}, \tag{45}$$

where $\delta_{p,q}$ is the Kronecker delta and $\Gamma(x)$ is the Gamma function.

From Eq. (33), one may write:

$$\big\{\mathbf{B}, \mathbf{b}^{(q)}\big\} = \beta_q, \tag{46}$$

where:

$$n^2 \beta_q = \int f_1^{(0)} \boldsymbol{\mathscr{C}}_1 \boldsymbol{\mathscr{C}}_1 : \mathbf{b}_1^{(q)}\, d\mathbf{c}_1 + \int f_2^{(0)} \boldsymbol{\mathscr{C}}_2 \boldsymbol{\mathscr{C}}_2 : \mathbf{b}_2^{(q)}\, d\mathbf{c}_2. \tag{47}$$

Integrating Eq. (47), one finds that:

$$\beta_1 = \frac{5}{2}\frac{n_1}{n^2}, \qquad \beta_{-1} = \frac{5}{2}\frac{n_2}{n^2}, \qquad \beta_q = 0 \quad (q \neq \pm 1). \tag{48}$$

Combining Eqs. (42)–(47) yields:

$$\sum_{\substack{p=-\infty \\ p\neq 0}}^{\infty} b_p b_{pq} = \beta_q, \tag{49}$$

where:

$$b_{pq} = \big\{\mathbf{b}^{(p)}, \mathbf{b}^{(q)}\big\}. \tag{50}$$

If values for b_{pq} are known, all of the b_p can be determined by solving the algebraic system of equations represented by Eq. (49). In the general matrix terminology of Chapman and Cowling, this is:

$$b_p = \lim_{m\to\infty} \big\{\beta_1 \mathscr{B}_{1p}^{(m)} + \beta_{-1} \mathscr{B}_{-1p}^{(m)}\big\} / \mathscr{B}^{(m)}, \tag{51}$$

where $\mathscr{B}^{(m)}$ is the determinant of the $2m \times 2m$ symmetric coefficient matrix of the system of Eq. (49) and $\mathscr{B}_{qp}^{(m)}$ is the cofactor of b_{qp} in the expansion of $\mathscr{B}^{(m)}$. Since, from Eq. (48) $\beta_q = 0$ $(q \neq \pm 1)$, this yields:

$$\{\mathbf{B}, \mathbf{B}\} = \sum_p b_p \big\{\mathbf{B}, \mathbf{b}^{(p)}\big\} = \beta_1 b_1 + \beta_{-1} b_{-1}, \tag{52}$$

which, in turn and still using general matrix terminology, yields the following expression for the viscosity of a binary gas mixture via Eqs. (40) and (51):

$$\mu = \frac{5}{2} kT \lim_{m\to\infty} \big\{x_1^2 \mathscr{B}_{11}^{(m)} + 2x_1x_2 \mathscr{B}_{1-1}^{(m)} + x_2^2 \mathscr{B}_{-1-1}^{(m)}\big\} / \mathscr{B}^{(m)}. \tag{53}$$

It can be shown that the successive approximations to μ as m is increased form a monotonically increasing series that converges to an exact value for μ in the limit as $m \to \infty$. Using Eqs. (40), (48), and (52), we note that the viscosity from Eq. (53) can be expressed in a more convenient form as:

$$\mu = p(x_1 b_1 + x_2 b_{-1}), \tag{54}$$

which is the form that we have used in the current work.

5. The bracket integrals

In order to complete the evaluation of the viscosity it is necessary to evaluate the bracket integrals defined in Eq. (34) for $\{\mathbf{b}^{(p)}, \mathbf{b}^{(q)}\}$. Hence, it is first necessary to be able to evaluate the square bracket integrals of Eqs. (35)–(37), specifically, $[\mathbf{b}_1^{(p)}, \mathbf{b}_1^{(q)}]_1$, $[\mathbf{b}_1^{(p)}, \mathbf{b}_1^{(q)}]_{12}$, and $[\mathbf{b}_1^{(p)}, \mathbf{b}_2^{(q)}]_{12}$. Completion of this task requires integration over all of the collision variables and, additionally, requires knowledge of the form of the intermolecular potential. However, the intermolecular potential affects only integrations over the variables, g and b, because they determine the scattering angle, χ. All six of the other integrals (there are eight total in the collision operator) can be evaluated in some manner without specific knowledge of the intermolecular potential.

The specific bracket integrals mentioned above that require evaluation are given explicitly in Chapman and Cowling but how they have been identified is not readily apparent in the text which focuses on very broad generalized expressions that can be difficult to interpret. In practice, the needed bracket integrals are most readily determined by considering Eqs. (25) and (26) directly. If these equations are expressed as:

$$n_1^2 I_1(\mathbf{B}_1) + n_1 n_2 I_{12}(\mathbf{B}_1) + n_1 n_2 I_{12}(\mathbf{B}_2) = f_1^{(0)} \overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, \tag{55}$$

$$n_2^2 I_2(\mathbf{B}_2) + n_1 n_2 I_{21}(\mathbf{B}_2) + n_1 n_2 I_{21}(\mathbf{B}_1) = f_2^{(0)} \overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}, \tag{56}$$

then one can insert Sonine polynomial approximations for the solutions, $\mathbf{B}_1$ and $\mathbf{B}_2$, can multiply through the equations with additional Sonine polynomials, and can then integrate the equations. By using the orthogonality of the Sonine polynomials as given in Eq. (45), this process will yield a set of simultaneous equations involving the matrix coefficients necessary to specify the solutions. The RHSs of the equations determine the constants, β_q, and the LHSs yield combinations of bracket integrals that correspond to the desired b_{pq} coefficients of the matrix used to determine the b_p values which, in turn, determine the viscosity via Eq. (54).

The easiest way to follow this process is via a low-order example. If one considers the Sonine polynomials used in the definition of the $\mathbf{b}_1^{(p)}$ and $\mathbf{b}_2^{(p)}$ Chapman–Enskog expansion tensors, $p = 1$ is the lowest order with $S_m^{(0)}(x) = 1$, such that $\mathbf{b}_1^{(1)} \equiv S_{5/2}^{(0)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1} = \overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}$ and $\mathbf{b}_2^{(-1)} \equiv S_{5/2}^{(0)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2} = \overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}$. Using only this order of approximation for $\mathbf{B}_1$ and $\mathbf{B}_2$, i.e. assuming that $\mathbf{B}_1 = b_1\mathbf{b}_1^{(1)} = b_1\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}$ and $\mathbf{B}_2 = b_{-1}\mathbf{b}_2^{(-1)} = b_{-1}\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}$, one may express Eqs. (55) and (56) as:

$$\begin{aligned} &n_1^2 I_1(\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1})b_1 + n_1 n_2 I_{12}(\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1})b_1 + n_1 n_2 I_{12}(\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2})b_{-1} \\ &\quad = f_1^{(0)}\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, \end{aligned} \tag{57}$$

$$\begin{aligned} &n_2^2 I_2(\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2})b_{-1} + n_1 n_2 I_{21}(\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2})b_{-1} + n_1 n_2 I_{21}(\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1})b_1 \\ &\quad = f_2^{(0)}\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}. \end{aligned} \tag{58}$$

Now, multiplying through Eq. (57) by $\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}$ and through Eq. (58) by $\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}$, one has:

$$\begin{aligned} &n_1^2\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1} : I_1(\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1})b_1 + n_1 n_2\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1} : I_{12}(\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1})b_1 \\ &\quad + n_1 n_2\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1} : I_{12}(\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2})b_{-1} = \overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1} : f_1^{(0)}\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, \end{aligned} \tag{59}$$

and:

$$\begin{aligned} &n_2^2\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2} : I_2(\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2})b_{-1} + n_1 n_2\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2} : I_{21}(\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2})b_{-1} \\ &\quad + n_1 n_2\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2} : I_{21}(\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1})b_1 = \overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2} : f_2^{(0)}\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}. \end{aligned} \tag{60}$$

From these, after integrating throughout Eqs. (59) and (60) in the manner of Eqs. (12) and (13) for I_i and I_{ij}, one can express Eqs. (59) and (60) in bracket integral notation as:

$$\begin{aligned} &\bigl(n_1^2[\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, \overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}]_1 + n_1 n_2[\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, \overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}]_{12}\bigr)b_1 \\ &\quad + n_1 n_2[\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, \overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}]_{12} b_{-1} \\ &\quad = \iint \overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1} : f_1^{(0)}\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}\, g\alpha_1\, \mathrm{d}\mathbf{e}'\, \mathrm{d}\mathbf{c}, \end{aligned} \tag{61}$$

and:

$$\begin{aligned} &n_1 n_2[\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}, \overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}]_{21} b_1 + \bigl(n_2^2[\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}, \overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}]_2 \\ &\quad + n_1 n_2[\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}, \overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}]_{21}\bigr)b_{-1} \\ &\quad = \iint \overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2} : f_2^{(0)}\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}\, g\alpha_2\, \mathrm{d}\mathbf{e}'\, \mathrm{d}\mathbf{c}. \end{aligned} \tag{62}$$

Given the way that the solution has been approached and the low order of this example, it is readily apparent that these equations (when divided through on both sides by n^2) must be equivalent to:

$$b_{11}b_1 + b_{1-1}b_{-1} = \beta_1, \tag{63}$$

$$b_{-11}b_1 + b_{-1-1}b_{-1} = \beta_{-1}, \tag{64}$$

where:

$$b_{11} = x_1^2[\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, \overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}]_1 + x_1 x_2[\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, \overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}]_{12}, \tag{65}$$

$$b_{1-1} = x_1 x_2[\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, \overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}]_{12}, \tag{66}$$

$$b_{-11} = x_1 x_2[\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}, \overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}]_{21}, \tag{67}$$

$$b_{-1-1} = x_2^2[\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}, \overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}]_2 + x_1 x_2[\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}, \overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}]_{21}, \tag{68}$$

and the integrals on the RHSs correspond to the β_q constants which, due to the orthogonality of the Sonine polynomials, are defined only in the current example where $q = \pm 1$ and are otherwise zero. Eqs. (63) and (64) can, of course, be rearranged to cast them into the ordered form that corresponds directly to the form that the more general, higher-order problem has been expressed in, i.e.:

$$\begin{bmatrix} b_{-1-1} & b_{-11} \\ b_{1-1} & b_{11} \end{bmatrix} \begin{bmatrix} b_{-1} \\ b_1 \end{bmatrix} = \begin{bmatrix} \beta_{-1} \\ \beta_1 \end{bmatrix}. \tag{69}$$

Thus, with the bracket integrals expressed in terms of the appropriate Sonine polynomials, the matrix elements of Eqs. (65)–(68) are simply:

$$\begin{aligned} b_{11} &= x_1^2\bigl[S_{5/2}^{(0)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(0)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}\bigr]_1 \\ &\quad + x_1 x_2\bigl[S_{5/2}^{(0)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(0)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}\bigr]_{12}, \end{aligned} \tag{70}$$

$$b_{1-1} = x_1 x_2\bigl[S_{5/2}^{(0)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(0)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}\bigr]_{12}, \tag{71}$$

$$b_{-11} = x_1 x_2\bigl[S_{5/2}^{(0)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}, S_{5/2}^{(0)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}\bigr]_{21}, \tag{72}$$

$$\begin{aligned} b_{-1-1} &= x_2^2\bigl[S_{5/2}^{(0)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}, S_{5/2}^{(0)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}\bigr]_2 \\ &\quad + x_1 x_2\bigl[S_{5/2}^{(0)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}, S_{5/2}^{(0)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}\bigr]_{21}, \end{aligned} \tag{73}$$

which, for higher-order expansions, become simply:

$$\begin{aligned} b_{pq} &= x_1^2\bigl[S_{5/2}^{(p-1)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(q-1)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}\bigr]_1 \\ &\quad + x_1 x_2\bigl[S_{5/2}^{(p-1)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(q-1)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}\bigr]_{12}, \end{aligned} \tag{74}$$

$$b_{p-q} = x_1 x_2\bigl[S_{5/2}^{(p-1)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(q-1)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}\bigr]_{12}, \tag{75}$$

$$b_{-pq} = x_1x_2\big[S_{5/2}^{(p-1)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}, S_{5/2}^{(q-1)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}\big]_{21}, \tag{76}$$

$$b_{-p-q} = x_2^2\big[S_{5/2}^{(p-1)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}, S_{5/2}^{(q-1)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}\big]_2 + x_1x_2\big[S_{5/2}^{(p-1)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}, S_{5/2}^{(q-1)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}\big]_{21}. \tag{77}$$

From the definitions of I_i and I_{ij} in Eqs. (12) and (13), it follows that Eqs. (76) and (77) are essentially identical to Eqs. (75) and (74), respectively, with the only difference being the interchange of the subscripts 1 and 2 representing the different components of the mixture. Thus, in general, the complete Chapman–Enskog solution for viscosity requires only the bracket integrals:

$$\begin{aligned}
&\big[S_{5/2}^{(p)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(q)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}\big]_1,\\
&\big[S_{5/2}^{(p)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(q)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}\big]_{12},\\
&\big[S_{5/2}^{(p)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(q)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}\big]_{12},
\end{aligned} \tag{78}$$

which are exactly those identified by Chapman and Cowling. Some further simplification of the problem can be obtained by noting from Chapman and Cowling that, in the limit of a simple (single) gas where $m_1 = m_2$, $n_1 = n_2 = n$, and $\alpha_{21} = \alpha_{12} = \alpha_1$, one has that $[F, G]_1 = [F_1, G_1 + G_2]_{12} = ([F_1, G_1]_{12} + [F_1, G_2]_{12})$ which, in the current problem, equates to:

$$\begin{aligned}
&\big[S_{5/2}^{(p)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(q)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}\big]_1\\
&\quad = \Big(\big[S_{5/2}^{(p)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(q)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}\big]_{12}\\
&\qquad + \big[S_{5/2}^{(p)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(q)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}\big]_{12}\Big)\Big|_{\substack{m_1=m_2\\ n_1=n_2=n\\ \alpha_{12}=\alpha_{21}=\alpha_1}}.
\end{aligned} \tag{79}$$

At this point, it still remains to perform the six integrations unrelated to the intermolecular potential model that is employed in order to complete the evaluation of the two necessary bracket integrals on the RHS of Eq. (79). For the relevant details of this integral evaluation process, one should refer to the text of Chapman and Cowling [1]. We note that Chapman and Cowling make use of the following definition of the Sonine polynomials:

$$(\mathrm{S}/s)^{m+1}\exp(-x\mathrm{S}) \equiv (1-s)^{-m-1}\exp\big(-xs/(1-s)\big) = \sum_{n=0}^{\infty} s^n S_m^{(n)}(x), \tag{80}$$

where $\mathrm{S} = s/(1-s)$ and, likewise, $\mathrm{T} = t/(1-t)$, are used to express the needed bracket integrals in terms of the coefficients of expansions in the arbitrarily introduced variables, s and t. Thus, it is possible after following Chapman and Cowling to determine that:

$$\begin{aligned}
&\big[S_{5/2}^{(p)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(q)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}\big]_{12}\\
&\quad = \mathrm{coeff}\big[s^pt^q\big]\bigg(\bigg(\frac{\mathrm{ST}}{st}\bigg)^{7/2}\pi^{-3}\\
&\qquad \times \iiint \big\{L_{12}(0) - L_{12}(\chi)\big\}gb\,\mathrm{d}b\,\mathrm{d}\varepsilon\,\mathrm{d}\boldsymbol{\mathscr{g}}\bigg),
\end{aligned} \tag{81}$$

and:

$$\begin{aligned}
&\big[S_{5/2}^{(p)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(q)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}\big]_{12}\\
&\quad = \mathrm{coeff}\big[s^pt^q\big]\bigg(\bigg(\frac{\mathrm{ST}}{st}\bigg)^{7/2}\pi^{-3}\\
&\qquad \times \iiint \big\{L_1(0) - L_1(\chi)\big\}gb\,\mathrm{d}b\,\mathrm{d}\varepsilon\,\mathrm{d}\boldsymbol{\mathscr{g}}\bigg),
\end{aligned} \tag{82}$$

where $\boldsymbol{\mathscr{g}} \equiv (m_0M_1M_2/2kT)^{1/2}\mathbf{g}_{21}$. Note that the retention of a single g in the integrands of Eqs. (81) and (82) (as opposed to $\mathscr{g}$) is not a typographical error but, rather, is the exact notation used by Chapman and Cowling. After integration over ε and the directions of $\boldsymbol{\mathscr{g}}$ (which changes the constants somewhat), one can express the χ-dependent portions of the RHS bracketed integrals of Eqs. (81) and (82) as:

$$\begin{aligned}
&\frac{3}{2}\bigg(\frac{\mathrm{ST}}{st}\bigg)^{7/2}(M_1M_2)^{-1}\pi^{-3/2}L_{12}(\chi)\\
&\quad = e^{-\mathscr{g}^2}\sum_r\sum_n\big\{2M_1M_2st\big(1-\cos(\chi)\big)\big\}^r\bigg(\frac{\mathscr{g}^{2r}}{r!}\bigg)(M_2s+M_1t)^n\\
&\qquad \times\bigg[(n+1)(n+2)S_{r+1/2}^{(n+2)}\big(\mathscr{g}^2\big)\\
&\qquad + 2(n+1)\mathscr{g}^2\big(1-\cos(\chi)\big)S_{r+3/2}^{(n+1)}\big(\mathscr{g}^2\big)\\
&\qquad + \mathscr{g}^4\bigg\{\big(1-\cos(\chi)\big)^2 - \frac{1}{2}\sin^2(\chi)\bigg\}S_{r+5/2}^{(n)}\big(\mathscr{g}^2\big)\bigg],
\end{aligned} \tag{83}$$

and:

$$\begin{aligned}
&\frac{3}{2}\bigg(\frac{\mathrm{ST}}{st}\bigg)^{7/2}\pi^{-3/2}L_1(\chi)\\
&\quad = e^{-\mathscr{g}^2}\sum_r\sum_n\big[st\big(M_1^2+M_2^2+2M_1M_2\cos(\chi)\big)\big]^r\bigg(\frac{\mathscr{g}^{2r}}{r!}\bigg)\\
&\qquad \times\big[M_2(s+t)-(M_2-M_1)st\big]^n\bigg[M_1^2(n+1)(n+2)S_{r+1/2}^{(n+2)}\big(\mathscr{g}^2\big)\\
&\qquad + 2(n+1)\mathscr{g}^2M_1\big(M_1+M_2\cos(\chi)\big)S_{r+3/2}^{(n+1)}\big(\mathscr{g}^2\big)\\
&\qquad + \mathscr{g}^4\bigg\{\big(M_1+M_2\cos(\chi)\big)^2 - \frac{1}{2}M_2^2\sin^2(\chi)\bigg\}S_{r+5/2}^{(n)}\big(\mathscr{g}^2\big)\bigg].
\end{aligned} \tag{84}$$

In both of these cases, the coefficient of $[s^pt^q]$ yields a polynomial in powers of $\mathscr{g}^2$ and $\cos(\chi)$ that is multiplied by $\exp(-\mathscr{g}^2)$ and in which each term is some function of the molecular masses via M_1 and M_2. The χ-independent portions of the RHS bracketed integrals of Eqs. (81) and (82) are obtained by the simple expedient of setting $\chi = 0$ in Eqs. (83) and (84) which yields overall terms in the combined polynomial involving $(1-\cos^\ell(\chi))$. Thus, it is possible to express Eqs. (81) and (82) as:

$$\begin{aligned}
&\big[S_{5/2}^{(p)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(q)}(\mathscr{C}_2^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_2\boldsymbol{\mathscr{C}}_2}\big]_{12}\\
&\quad = \frac{16}{3}\pi^{1/2}M_2^{p+1}M_1^{q+1}\iint\exp\big(-\mathscr{g}^2\big)\\
&\qquad \times\sum_{r,\ell}(B_{pqr\ell})\mathscr{g}^{2r+2}\big(1-\cos^\ell(\chi)\big)\mathrm{g}b\,\mathrm{d}b\,\mathrm{d}\mathscr{g}\\
&\quad = \frac{16}{3}M_2^{p+1}M_1^{q+1}\sum_{r,\ell}(B_{pqr\ell})\Omega_{12}^{(\ell)}(r),
\end{aligned} \tag{85}$$

and:

$$\begin{aligned}
&\big[S_{5/2}^{(p)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}, S_{5/2}^{(q)}(\mathscr{C}_1^2)\overset{\circ}{\boldsymbol{\mathscr{C}}_1\boldsymbol{\mathscr{C}}_1}\big]_{12}\\
&\quad = \frac{16}{3}\pi^{1/2}\iint\exp\big(-\mathscr{g}^2\big)\sum_{r,\ell}(B'_{pqr\ell})\mathscr{g}^{2r+2}\big(1-\cos^\ell(\chi)\big)\mathrm{g}b\,\mathrm{d}b\,\mathrm{d}\mathscr{g}\\
&\quad = \frac{16}{3}\sum_{r,\ell}(B'_{pqr\ell})\Omega_{12}^{(\ell)}(r),
\end{aligned} \tag{86}$$

where the omega integrals are defined as:

$$\Omega_{12}^{(\ell)}(r) \equiv \bigg(\frac{kT}{2\pi m_0M_1M_2}\bigg)^{1/2}\int_0^\infty \exp\big(-\mathscr{g}^2\big)\mathscr{g}^{2r+3}\phi_{12}^{(\ell)}\,\mathrm{d}\mathscr{g}, \tag{87}$$

with:

$$\phi_{12}^{(\ell)} \equiv 2\pi \int_0^{\pi} \big(1-\cos^{\ell}(\chi)\big) b\,\mathrm{d}b. \tag{88}$$

As stated by Chapman and Cowling [1]:

> "Explicit expressions for $[B_{pqr\ell}$ and $B'_{pqr\ell}]$ can be obtained from [Eqs. (83) and (84)] using [Eq. (44)] for $S_m^{(n)}(x)$. In view of the complication of these expressions it is, however, better in practice to calculate any desired values of $[B_{pqr\ell}$ and $B'_{pqr\ell}]$ directly from [Eqs. (83) and (84)]."

We have explored this approach as suggested by Chapman and Cowling using *Mathematica*® up to order 60 and are reporting our order 60 viscosity results here. We have used *Mathematica*® for this work for its versatility in permitting both symbolic and high-precision computations although we note that other programming environments (such as Fortran, C++, etc.) could also have been used with additional effort. In all of our expressions, we have retained the fully general dependence of the expressions on the mole fractions, the molecular masses, and the models of the intermolecular potential that can be employed. For example, from Eq. (49), since there exists symmetry in the off-diagonal elements such that $b_{pq} = b_{qp}$, one can write the following matrix equation for order 1 that we arrived at previously in our low-order example:

$$\begin{bmatrix} b_{-1-1} & b_{-11} \\ b_{1-1} & b_{11} \end{bmatrix} \begin{bmatrix} b_{-1} \\ b_1 \end{bmatrix} = \begin{bmatrix} \beta_{-1} \\ \beta_1 \end{bmatrix}. \tag{89}$$

Here, we have explicitly determined the general expressions for the b_{pq} coefficients to be:

$$\begin{aligned} b_{-1-1} &= x_2^2\big\{4\Omega_2^{(2)}(2)\big\} \\ &\quad + x_1x_2\left\{\frac{80}{3}(M_1M_2)\Omega_{12}^{(1)}(1) + 8\big(M_1^2\big)\Omega_{12}^{(2)}(2)\right\}, \end{aligned} \tag{90}$$

$$b_{-11} = x_1x_2\left\{-\frac{80}{3}(M_1M_2)\Omega_{12}^{(1)}(1) + 8(M_1M_2)\Omega_{12}^{(2)}(2)\right\}, \tag{91}$$

$$b_{1-1} = x_1x_2\left\{-\frac{80}{3}(M_1M_2)\Omega_{12}^{(1)}(1) + 8(M_1M_2)\Omega_{12}^{(2)}(2)\right\}, \tag{92}$$

$$\begin{aligned} b_{11} &= x_1^2\big\{4\Omega_1^{(2)}(2)\big\} \\ &\quad + x_1x_2\left\{\frac{80}{3}(M_1M_2)\Omega_{12}^{(1)}(1) + 8\big(M_2^2\big)\Omega_{12}^{(2)}(2)\right\}. \end{aligned} \tag{93}$$

The symmetry in the off-diagonal elements is obvious, with $b_{-11} = b_{1-1}$. Likewise, for order 2, Eq. (49) plus symmetry gives the following matrix equation:

$$\begin{bmatrix} b_{-2-2} & b_{-2-1} & b_{-21} & b_{-22} \\ b_{-1-2} & b_{-1-1} & b_{-11} & b_{-12} \\ b_{1-2} & b_{1-1} & b_{11} & b_{12} \\ b_{2-2} & b_{2-1} & b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} b_{-2} \\ b_{-1} \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \beta_{-1} \\ \beta_1 \\ 0 \end{bmatrix}, \tag{94}$$

and we have determined the general expressions for the b_{pq} coefficients to be:

$$\begin{aligned} b_{-2-2} &= x_2^2\left\{\frac{301}{12}\Omega_2^{(2)}(2) - 7\Omega_2^{(2)}(3) + \Omega_2^{(2)}(4)\right\} \\ &\quad + x_1x_2\left\{\frac{1540}{3}\left(\frac{4}{11}M_1M_2^3 + \frac{7}{11}M_1^3M_2\right)\Omega_{12}^{(1)}(1) \right. \\ &\quad - \frac{784}{3}\big(M_1^3M_2\big)\Omega_{12}^{(1)}(2) + \frac{128}{3}\big(M_1^3M_2\big)\Omega_{12}^{(1)}(3) \\ &\quad + \frac{602}{3}\left(\frac{22}{43}M_1^2M_2^2 + \frac{21}{43}M_1^4\right)\Omega_{12}^{(2)}(2) - 56\big(M_1^4\big)\Omega_{12}^{(2)}(3) \\ &\quad \left. + 8\big(M_1^4\big)\Omega_{12}^{(2)}(4) + 16\big(M_1^3M_2\big)\Omega_{12}^{(3)}(3)\right\}, \end{aligned} \tag{95}$$

$$\begin{aligned} b_{-2-1} = b_{-1-2} &= x_2^2\big\{7\Omega_2^{(2)}(2) - 2\Omega_2^{(2)}(3)\big\} \\ &\quad + x_1x_2\left\{\frac{280}{3}\big(M_1^2M_2\big)\Omega_{12}^{(1)}(1) \right. \\ &\quad - \frac{112}{3}\big(M_1^2M_2\big)\Omega_{12}^{(1)}(2) + 28\big(M_1^3\big)\Omega_{12}^{(2)}(2) \\ &\quad \left. - 8\big(M_1^3\big)\Omega_{12}^{(2)}(3)\right\}, \end{aligned} \tag{96}$$

$$\begin{aligned} b_{-21} = b_{1-2} &= x_1x_2\left\{-\frac{280}{3}\big(M_1^2M_2\big)\Omega_{12}^{(1)}(1) + \frac{112}{3}\big(M_1^2M_2\big)\Omega_{12}^{(1)}(2) \right. \\ &\quad \left. + 28\big(M_1^2M_2\big)\Omega_{12}^{(2)}(2) - 8\big(M_1^2M_2\big)\Omega_{12}^{(2)}(3)\right\}, \end{aligned} \tag{97}$$

$$\begin{aligned} b_{-22} = b_{2-2} &= x_1x_2\left\{-\frac{1540}{3}\big(M_1^2M_2^2\big)\Omega_{12}^{(1)}(1) + \frac{784}{3}\big(M_1^2M_2^2\big)\Omega_{12}^{(1)}(2) \right. \\ &\quad - \frac{128}{3}\big(M_1^2M_2^2\big)\Omega_{12}^{(1)}(3) + \frac{602}{3}\big(M_1^2M_2^2\big)\Omega_{12}^{(2)}(2) \\ &\quad - 56\big(M_1^2M_2^2\big)\Omega_{12}^{(2)}(3) + 8\big(M_1^2M_2^2\big)\Omega_{12}^{(2)}(4) \\ &\quad \left. - 16\big(M_1^2M_2^2\big)\Omega_{12}^{(3)}(3)\right\}, \end{aligned} \tag{98}$$

$$\begin{aligned} b_{-1-1} &= x_2^2\big\{4\Omega_2^{(2)}(2)\big\} \\ &\quad + x_1x_2\left\{\frac{80}{3}(M_1M_2)\Omega_{12}^{(1)}(1) + 8\big(M_1^2\big)\Omega_{12}^{(2)}(2)\right\}, \end{aligned} \tag{99}$$

$$b_{-11} = b_{1-1} = x_1x_2\left\{-\frac{80}{3}(M_1M_2)\Omega_{12}^{(1)}(1) + 8(M_1M_2)\Omega_{12}^{(2)}(2)\right\}, \tag{100}$$

$$\begin{aligned} b_{-12} = b_{2-1} &= x_1x_2\left\{-\frac{280}{3}\big(M_1M_2^2\big)\Omega_{12}^{(1)}(1) + \frac{112}{3}\big(M_1M_2^2\big)\Omega_{12}^{(1)}(2) \right. \\ &\quad \left. + 28\big(M_1M_2^2\big)\Omega_{12}^{(2)}(2) - 8\big(M_1M_2^2\big)\Omega_{12}^{(2)}(3)\right\}, \end{aligned} \tag{101}$$

$$b_{11} = x_1^2\big\{4\Omega_1^{(2)}(2)\big\} + x_1x_2\left\{\frac{80}{3}(M_1M_2)\Omega_{12}^{(1)}(1) + 8\big(M_2^2\big)\Omega_{12}^{(2)}(2)\right\}, \tag{102}$$

$$\begin{aligned} b_{12} = b_{21} &= x_1^2\big\{7\Omega_1^{(2)}(2) - 2\Omega_1^{(2)}(3)\big\} + x_1x_2\left\{\frac{280}{3}\big(M_1M_2^2\big)\Omega_{12}^{(1)}(1) \right. \\ &\quad - \frac{112}{3}\big(M_1M_2^2\big)\Omega_{12}^{(1)}(2) + 28\big(M_2^3\big)\Omega_{12}^{(2)}(2) \\ &\quad \left. - 8\big(M_2^3\big)\Omega_{12}^{(2)}(3)\right\}, \end{aligned} \tag{103}$$

$$\begin{aligned} b_{22} &= x_1^2\left\{\frac{301}{12}\Omega_1^{(2)}(2) - 7\Omega_1^{(2)}(3) + \Omega_1^{(2)}(4)\right\} \\ &\quad + x_1x_2\left\{\frac{1540}{3}\left(\frac{4}{11}M_1^3M_2 + \frac{7}{11}M_1M_2^3\right)\Omega_{12}^{(1)}(1) \right. \\ &\quad - \frac{784}{3}\big(M_1M_2^3\big)\Omega_{12}^{(1)}(2) + \frac{128}{3}\big(M_1M_2^3\big)\Omega_{12}^{(1)}(3) \\ &\quad + \frac{602}{3}\left(\frac{22}{43}M_1^2M_2^2 + \frac{21}{43}M_2^4\right)\Omega_{12}^{(2)}(2) - 56\big(M_2^4\big)\Omega_{12}^{(2)}(3) \\ &\quad \left. + 8\big(M_2^4\big)\Omega_{12}^{(2)}(4) + 16\big(M_1M_2^3\big)\Omega_{12}^{(3)}(3)\right\}. \end{aligned} \tag{104}$$

In Eqs. (95)–(104), the off-diagonal symmetry has been expressed explicitly. While we could, certainly, go even higher in order detailing explicit expressions for the matrix coefficients, the expressions rapidly become unwieldy in analytical form. If one wishes to verify our order 2 expressions or if additional explicit expressions for the order 3 matrix coefficients are desired, we note that they can be constructed directly from the relevant bracket integral expressions which have been reported previously in the literature [20,21]. In producing analytical order 60 results computationally we have, of course, gone far beyond the hand calculated orders 2 and 3 reported in the previous literature and are in essential agreement with those bracket integral results except for an apparent typographical error in Ref. [21] where one coefficient in the bracket integral corresponding to (in our notation) $[S^{(2)}_{5/2}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\overset{\circ}{\boldsymbol{\mathscr{C}}}_1, S^{(1)}_{5/2}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\overset{\circ}{\boldsymbol{\mathscr{C}}}_2]_{12}$ for the omega-integral $\Omega^{(2)}_{ij}(2)$ should be changed from $\frac{4095}{16}$ to $\frac{4095}{32}$. The bracket integrals appropriate to higher orders have not been previously reported to our knowledge. In going computationally to higher orders, all of the previously generated expressions are retained and need not be recomputed. Most importantly, regardless of the order of the expansion used, the dependencies of the matrix coefficients on mole fractions and molecular masses are carried through in general form. Molecular diameters and the specific intermolecular potential model used are also carried through in general form via the omega integrals which may also be defined in terms of the quantity, σ_{12}. This quantity is, in purely general terms, only a convenient, arbitrarily chosen length in the range of b. Thus, it is often convenient to express the omega-integrals as [1]:

$$\Omega^{(\ell)}_{12}(r) = \frac{1}{2}\sigma_{12}^2\left(\frac{2\pi kT}{m_0 M_1 M_2}\right)^{1/2} W^{(\ell)}_{12}(r), \tag{105}$$

where:

$$\begin{aligned} W^{(\ell)}_{12}(r) &\equiv \int_0^\infty \exp(-g^2) g^{2r+2} \int_0^\pi \left(1-\cos^\ell(\chi)\right)\left(\frac{b}{\sigma_{12}}\right) \mathrm{d}\left(\frac{b}{\sigma_{12}}\right) \mathrm{d}\left(g^2\right) \\ &= 2\int_0^\infty \exp(-g^2) g^{2r+3} \int_0^\pi \left(1-\cos^\ell(\chi)\right)\left(\frac{b}{\sigma_{12}}\right) \mathrm{d}\left(\frac{b}{\sigma_{12}}\right) \mathrm{d}g, \end{aligned} \tag{106}$$

and where the corresponding simple gas expressions are:

$$\Omega^{(\ell)}_{1}(r) = \sigma_1^2\left(\frac{\pi kT}{m_1}\right)^{1/2} W^{(\ell)}_{1}(r), \tag{107}$$

with:

$$W^{(\ell)}_{1}(r) = 2\int_0^\infty \exp(-g^2) g^{2r+3} \int_0^\pi \left(1-\cos^\ell(\chi)\right)\left(\frac{b}{\sigma_1}\right) \mathrm{d}\left(\frac{b}{\sigma_1}\right) \mathrm{d}g, \tag{108}$$

and:

$$\Omega^{(\ell)}_{2}(r) = \sigma_2^2\left(\frac{\pi kT}{m_2}\right)^{1/2} W^{(\ell)}_{2}(r), \tag{109}$$

with:

$$W^{(\ell)}_{2}(r) = 2\int_0^\infty \exp(-g^2) g^{2r+3} \int_0^\pi \left(1-\cos^\ell(\chi)\right)\left(\frac{b}{\sigma_2}\right) \mathrm{d}\left(\frac{b}{\sigma_2}\right) \mathrm{d}g. \tag{110}$$

In Eqs. (105)–(110), σ_1 and σ_2 are arbitrary scale lengths associated with collisions between like molecules of type 1 and type 2, respectively, while σ_{12} is associated with collisions between unlike molecules of types 1 and 2. These scale lengths are commonly associated with some concept of the molecular diameters depending upon the specific details of the intermolecular potential model that is employed. In the current work, we are reporting results for the case of rigid-sphere molecules because the form of the potential model allows the omega integrals to be evaluated analytically eliminating the need for any numerical integrations in the current work. Some of the details of this potential model are described in the following section.

6. The case of rigid-sphere molecules

For the specific case of a binary, rigid-sphere, gas mixture, one has an intermolecular potential model of the form [22]:

$$U(r) = \begin{cases} \infty, & r \leqslant \sigma_{12}, \\ 0, & r > \sigma_{12}, \end{cases} \tag{111}$$

where $\sigma_{12} = \frac{1}{2}(\sigma_1 + \sigma_2)$, and σ_1 and σ_2 are the actual diameters of the colliding spherical molecules. Under the rigid-sphere assumption, one then has the collisional relationships $b = \sigma_{12}\cos(\frac{1}{2}\chi)$ and $b\,\mathrm{d}b = -\frac{1}{4}\sigma_{12}^2\sin(\chi)\,\mathrm{d}\chi$. Using these in Eqs. (105) and (106) yields:

$$W^{(\ell)}_{12}(r) = \frac{1}{4}\left[2 - \frac{1}{\ell+1}\left(1+(-1)^\ell\right)\right](r+1)!, \tag{112}$$

such that:

$$\Omega^{(\ell)}_{12}(r) = \frac{1}{2}\sigma_{12}^2\left(\frac{2\pi kT}{m_0 M_1 M_2}\right)^{1/2}\frac{1}{4}\left[2 - \frac{1}{\ell+1}\left(1+(-1)^\ell\right)\right](r+1)!, \tag{113}$$

and, since $W^{(\ell)}_1(r) = W^{(\ell)}_2(r) = W^{(\ell)}_{12}(r)$, the corresponding simple-gas expressions:

$$\Omega^{(\ell)}_{1}(r) = \sigma_1^2\left(\frac{\pi kT}{m_1}\right)^{1/2}\frac{1}{4}\left[2 - \frac{1}{\ell+1}\left(1+(-1)^\ell\right)\right](r+1)!, \tag{114}$$

and:

$$\Omega^{(\ell)}_{2}(r) = \sigma_2^2\left(\frac{\pi kT}{m_2}\right)^{1/2}\frac{1}{4}\left[2 - \frac{1}{\ell+1}\left(1+(-1)^\ell\right)\right](r+1)!. \tag{115}$$

All of these omega integrals are readily evaluated for the purpose of the current work and, thus, one has the following simplified, rigid-sphere, matrix elements for the order 2 approximation:

$$\begin{aligned} b_{-2-2} = x_2^2&\left\{\sqrt{\frac{\pi kT}{m_2}}\frac{205}{6}\sigma_2^2\right\} \\ &+ x_1x_2\left\{\sqrt{\frac{2\pi kT}{m_0 M_1 M_2}}\left(34M_1^4 + \frac{370}{3}M_1^3M_2\right.\right. \\ &\left.\left.+ \frac{308}{3}M_1^2M_2^2 + \frac{280}{3}M_1M_2^3\right)\sigma_{12}^2\right\}, \end{aligned} \tag{116}$$

$$\begin{aligned} b_{-2-1} = b_{-1-2} = x_2^2\left\{-\sqrt{\frac{\pi kT}{m_2}}2\sigma_2^2\right\} + x_1x_2\left\{-\sqrt{\frac{2\pi kT}{m_0 M_1 M_2}}\left(4M_1^3\right.\right. \\ \left.\left.+ \frac{28}{3}M_1^2M_2\right)\sigma_{12}^2\right\}, \end{aligned} \tag{117}$$

$$b_{-21} = b_{1-2} = x_1x_2\left\{\sqrt{\frac{2\pi kT}{m_0 M_1 M_2}}\frac{16}{3}M_1^2M_2\sigma_{12}^2\right\}, \tag{118}$$

$$b_{-22} = b_{2-2} = x_1x_2\left\{-\sqrt{\frac{2\pi kT}{m_0 M_1 M_2}}80M_1^2M_2^2\sigma_{12}^2\right\}, \tag{119}$$

$$b_{-1-1} = x_2^2\left\{\sqrt{\frac{\pi kT}{m_2}}8\sigma_2^2\right\} + x_1x_2\left\{\sqrt{\frac{2\pi kT}{m_0M_1M_2}}\left(8M_1^2 + \frac{40}{3}M_1M_2\right)\sigma_{12}^2\right\}, \tag{120}$$

$$b_{-11} = b_{1-1} = x_1x_2\left\{-\sqrt{\frac{2\pi kT}{m_0M_1M_2}}\frac{16}{3}M_1M_2\sigma_{12}^2\right\}, \tag{121}$$

$$b_{-12} = b_{2-1} = x_1x_2\left\{\sqrt{\frac{2\pi kT}{m_0M_1M_2}}\frac{16}{3}M_1M_2^2\sigma_{12}^2\right\}, \tag{122}$$

$$b_{11} = x_1^2\left\{\sqrt{\frac{\pi kT}{m_1}}8\sigma_1^2\right\} + x_1x_2\left\{\sqrt{\frac{2\pi kT}{m_0M_1M_2}}\left(8M_2^2 + \frac{40}{3}M_1M_2\right)\sigma_{12}^2\right\}, \tag{123}$$

$$b_{12} = b_{21} = x_1^2\left\{-\sqrt{\frac{\pi kT}{m_1}}2\sigma_1^2\right\} + x_1x_2\left\{-\sqrt{\frac{2\pi kT}{m_0M_1M_2}}\left(4M_2^3 + \frac{28}{3}M_1M_2^2\right)\sigma_{12}^2\right\}, \tag{124}$$

$$b_{22} = x_1^2\left\{\sqrt{\frac{\pi kT}{m_1}}\frac{205}{6}\sigma_1^2\right\} + x_1x_2\left\{\sqrt{\frac{2\pi kT}{m_0M_1M_2}}\left(34M_2^4 + \frac{370}{3}M_1M_2^3 + \frac{308}{3}M_1^2M_2^2 + \frac{280}{3}M_1^3M_2\right)\sigma_{12}^2\right\}. \tag{125}$$

In our results, this methodology is continued to arbitrary order with the full dependencies of the matrix elements on the molecular masses, mole fractions, and molecular diameters being retained explicitly up to the final point of actual evaluation via matrix inversion. As indicated previously, adaptation of this work to more realistic potential models is straightforward since the potential model is present in the bracket integral expressions via the omega integrals.

7. Results

The quantities of major interest in the present work are the viscosity:

$$\mu = p(x_1b_1 + x_2b_{-1}), \tag{126}$$

and the Chapman–Enskog viscosity solutions:

$$\mathbb{B}_1(\mathscr{C}_1) = \sum_{p=1}^{+\infty} b_p S_{5/2}^{(p-1)}\left(\mathscr{C}_1^2\right)\mathscr{C}_1^2, \tag{127}$$

$$\mathbb{B}_2(\mathscr{C}_2) = \sum_{p=-1}^{-\infty} b_p S_{5/2}^{(-p-1)}\left(\mathscr{C}_2^2\right)\mathscr{C}_2^2. \tag{128}$$

Here, note that b_1 and b_{-1} are actually integrals on $\mathbb{B}_1$ and $\mathbb{B}_2$ and, thus, may be expressed as:

$$b_1 = \frac{16}{15\sqrt{\pi}}\int_0^\infty \exp\left(-\mathscr{C}_1^2\right)\mathscr{C}_1^4\mathbb{B}_1(\mathscr{C}_1)\,\mathrm{d}\mathscr{C}_1, \tag{129}$$

and:

$$b_{-1} = \frac{16}{15\sqrt{\pi}}\int_0^\infty \exp\left(-\mathscr{C}_2^2\right)\mathscr{C}_2^4\mathbb{B}_2(\mathscr{C}_2)\,\mathrm{d}\mathscr{C}_2. \tag{130}$$

Note that $\mathbb{B}_1$, $\mathbb{B}_2$, b_1, b_{-1}, and μ are all dependent upon x_1, x_2, m_1, m_2, σ_1, σ_2, and temperature, T, although these dependencies are not displayed explicitly in the above equations. In the mth order of the expansion, we write these as:

$$[\mu]_m = p\left(x_1b_1^{(m)} + x_2b_{-1}^{(m)}\right), \tag{131}$$

$$\mathbb{B}_1^{(m)}(\mathscr{C}_1) = \sum_{p=1}^{m} b_p^{(m)} S_{5/2}^{(p-1)}\left(\mathscr{C}_1^2\right)\mathscr{C}_1^2, \tag{132}$$

$$\mathbb{B}_2^{(m)}(\mathscr{C}_2) = \sum_{p=1}^{m} b_{-p}^{(m)} S_{5/2}^{(p-1)}\left(\mathscr{C}_2^2\right)\mathscr{C}_2^2, \tag{133}$$

$$b_1^{(m)} = \frac{16}{15\sqrt{\pi}}\int_0^\infty \exp\left(-\mathscr{C}_1^2\right)\mathscr{C}_1^4\mathbb{B}_1^{(m)}(\mathscr{C}_1)\,\mathrm{d}\mathscr{C}_1, \tag{134}$$

and:

$$b_{-1}^{(m)} = \frac{16}{15\sqrt{\pi}}\int_0^\infty \exp\left(-\mathscr{C}_2^2\right)\mathscr{C}_2^4\mathbb{B}_2^{(m)}(\mathscr{C}_2)\,\mathrm{d}\mathscr{C}_2. \tag{135}$$

As a part of this work, we have carried out the following sets of calculations:

(i) We have conducted a comparison of our results from the current work with the results previously reported by Takata et al. [18]. To conduct this comparison, we have adapted our results so as to present them using the same non-dimensionalization scheme employed by Takata et al. and have then plotted our adapted results in graphical form with the same scaling and for the same set of virtual gas mixtures reported by Takata et al. Note that the graphical results reported here are from the current work. The results of Takata et al. are indistinguishable by eye from the current results and have not been re-reported here.

(ii) We have conducted a comparison of our results from the current work with the results previously reported by Garcia and Siewert [19]. In this comparison, we have again adapted our results so as to present them using the same non-dimensionalization scheme employed by Garcia and Siewert which is in part the same as that used by Takata et al. [18]. However, Garcia and Siewert presented their results numerically rather than graphically for selected real-gas mixtures and we have included our similarly adapted numerical results corresponding to the same real-gas mixtures. Additionally, Garcia and Siewert presented numerical results for the non-dimensionalized quantities, $\varepsilon_{p,1}$, $\varepsilon_{p,2}$, $\widehat{B}_1$, and $\widehat{B}_2$ (defined below) which we have also computed in the same non-dimensionalized forms for comparison purposes. Here, we have included some of our actual numerical values from orders 58–60 which allows the degree of convergence of our results to be explicitly assessed in each case.

(iii) Using the non-dimensionalization/normalization scheme of Chapman and Cowling [1] which normalizes viscosity relative to the first-order approximate results, we have obtained a comprehensive set of order 60 results for all binary mixtures of the noble gases He, Ne, Ar, Kr, and Xe. The two cases considered by Garcia and Siewert [19] are a subset of these collected results which are presented numerically for orders 1 (un-normalized) and 60 (normalized to order 1). Additionally, we present extrapolated limiting values of the viscosities (designated by a subscript "Wynn") that have been obtained by applying the *Mathematica*® function `SequenceLimit` to the sequence of normalized viscosity results corresponding to orders 1 through 60.

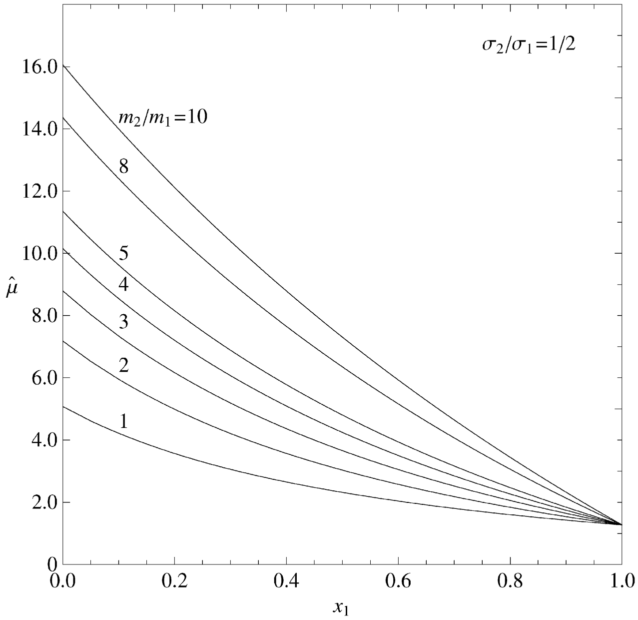

Fig. 1. The $\hat{\mu}$ functions calculated in this work for comparison with the values reported by Takata et al. [18] for $\sigma_2/\sigma_1 = 1/2$.

In the work of Takata et al. [18], the authors have considered cases involving a selection of "virtual" gas mixtures for which the size and mass ratios of the constituents are general values only and do not reflect the sizes and masses of specific gas constituents. The size ratios that have been considered in this work include $\sigma_2/\sigma_1 = \frac{1}{2}, 1, 2$ and the mass ratios that have been considered include $m_2/m_1 = 1, 2, 3, 4, 5, 8, 10$. Mole fractions are specified by $x_1 \in (0, 1)$ with $x_1 + x_2 = 1$. The authors define a non-dimensional viscosity as:

$$\hat{\mu} = f_1\mu, \tag{136}$$

where:

$$f_1 = \left(\frac{\sqrt{m_1kT}}{\sigma_1^2}\frac{1}{4\sqrt{\pi}}\right)^{-1}, \tag{137}$$

and, thus, $\hat{\mu}$ depends only upon the masses, m_1 and m_2, the molecular diameters, σ_1 and σ_2, and the mole fractions, x_1 and x_2. Dependence upon the temperature, T, has been totally eliminated for rigid-sphere molecules which is quite convenient because the rigid-sphere model is known to exhibit an unrealistic temperature dependence. The authors have presented their results graphically and have not given their precise numerical results explicitly although our comparison results indicate that they have obtained precise results for the cases studied. We have reported our results graphically, in the same format and for exactly the same cases considered by Takata et al., in Figs. 1–3 where our reported results conform to the same non-dimensionalization scheme described in Eqs. (136)–(137) and which we note are visually indistinguishable from the graphical results of Takata et al. which have not been explicitly reproduced here.

In the work of Garcia and Siewert [19], the authors have considered Ne:Ar and He:Xe gas mixtures. They have reported values of the non-dimensionalized quantities $\hat{\mu}$, $\varepsilon_{p,1}$, $\varepsilon_{p,2}$, $\widehat{B}_1$, and $\widehat{B}_2$ that are related to our results in the following manner:

$$\hat{\mu} = f_1\mu = 2^{3/2}\left(\frac{\sigma_1}{x_1\sigma_1 + x_2\sigma_2}\right)^2(x_1\varepsilon_{p,1} + x_2\varepsilon_{p,2}), \tag{138}$$

$$\varepsilon_{p,1} = f_2b_1, \tag{139}$$

$$\varepsilon_{p,2} = f_2b_{-1}, \tag{140}$$

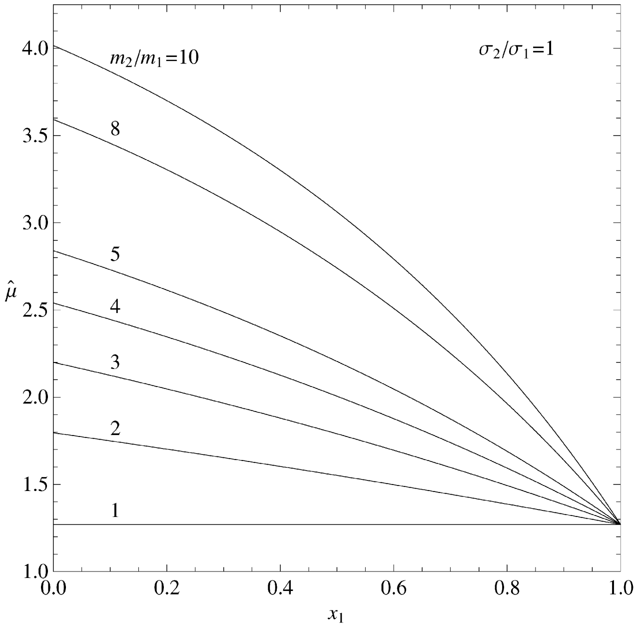

Fig. 2. The $\hat{\mu}$ functions calculated in this work for comparison with the values reported by Takata et al. [18] for $\sigma_2/\sigma_1 = 1$.

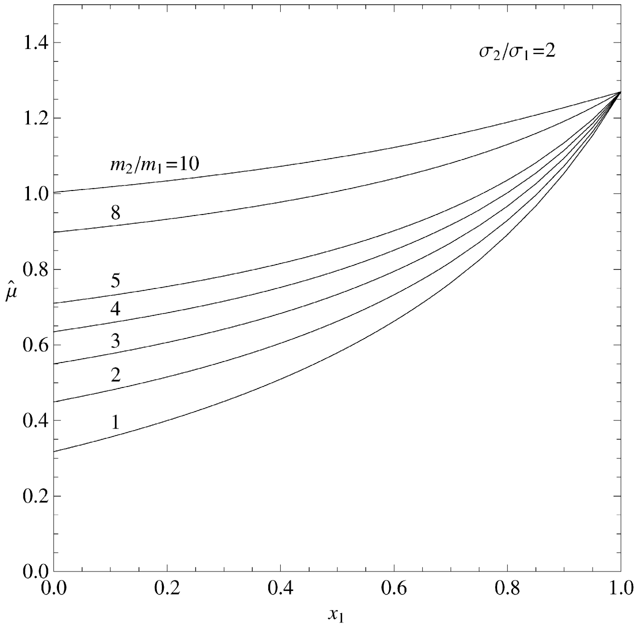

Fig. 3. The $\hat{\mu}$ functions calculated in this work for comparison with the values reported by Takata et al. [18] for $\sigma_2/\sigma_1 = 2$.

$$\widehat{B}_1 = f_2\mathbb{B}_1, \tag{141}$$

$$\widehat{B}_2 = f_2\mathbb{B}_2, \tag{142}$$

where f_1 is the same non-dimensionalization used by Takata et al. [18] in Eq. (136) and:

$$f_2 = f_1 2^{-3/2} p\left(\frac{\sigma_1}{x_1\sigma_1 + x_2\sigma_2}\right)^{-2}. \tag{143}$$

We have, again, adapted our results in a corresponding manner in order to conduct a comparison with Garcia and Siewert in their notation and this comparison is given in numerical form in Tables 1–5 for selected cases. Note that for the Ne:Ar mixture, Garcia and Siewert used molecular weights of 20.183 and 39.948 for Ne

Table 1
Values of $\varepsilon_{p,1}$, $\varepsilon_{p,1}$, and $\hat{\mu}$ presented in the dimensionless form used by Garcia and Siewert [19] and computed using the same molecular masses and sizes that they used for a binary, rigid-sphere, gas mixture of Ne:Ar with the current order 58, 59, and 60 results arbitrarily truncated at 20 significant figures.

	Garcia and Siewert [19]			Present work			
x_1	$\varepsilon_{p,1}$	$\varepsilon_{p,2}$	$\hat{\mu}^{\dagger}$	m	$\varepsilon_{p,1}$	$\varepsilon_{p,2}$	$\hat{\mu}$
0.0	0.5595420	0.6317248	0.903862	58	0.5595419676692080135	0.63172475587517940212	0.90386225002530434319
				59	0.5595419676692087027	0.63172475587517940213	0.90386225002530434320
				60	0.5595419676692092190	0.63172475587517940213	0.90386225002530434321
0.1	0.5476674	0.6176388	0.926428	58	0.5476674230030515696	0.61763878392026992253	0.92642788323526232109
				59	0.5476674230030519489	0.61763878392026989958	0.92642788323526234729
				60	0.5476674230030522315	0.61763878392026988248	0.92642788323526236680
0.2	0.5359138	0.6036851	0.951027	58	0.5359137600871278461	0.60368509577823719786	0.95102658536050854817
				59	0.5359137600871280432	0.60368509577823717168	0.95102658536050857795
				60	0.5359137600871281891	0.60368509577823715230	0.95102658536050859997
0.3	0.5242967	0.5898802	0.977937	58	0.5242967335652768020	0.58988023823505222751	0.97793688289859194166
				59	0.5242967335652768976	0.58988023823505220635	0.97793688289859196545
				60	0.5242967335652769678	0.58988023823505219079	0.97793688289859198292
0.4	0.5128351	0.5762439	1.00749	58	0.5128351292637218386	0.57624393073746343774	1.0074903509531462084
				59	0.5128351292637218813	0.57624393073746342354	1.0074903509531462240
				60	0.5128351292637219124	0.57624393073746341318	1.0074903509531462354
0.5	0.5015515	0.5627999	1.04008	58	0.5015515418963773753	0.56279988306568113001	1.0400848946575599796
				59	0.5015515418963773925	0.56279988306568112178	1.0400848946575599884
				60	0.5015515418963774049	0.56279988306568111583	1.0400848946575599947
0.6	0.4904734	0.5495769	1.0762	58	0.4904734073043349838	0.54957688246219324868	1.0762022481161614336
				59	0.4904734073043349898	0.54957688246219324456	1.0762022481161614378
				60	0.4904734073043349942	0.54957688246219324160	1.0762022481161614407
0.7	0.4796344	0.5366103	1.11643	58	0.4796343927427824630	0.53661026065287905164	1.1164313496741517441
				59	0.4796343927427824648	0.53661026065287904989	1.1164313496741517458
				60	0.4796343927427824660	0.53661026065287904865	1.1164313496741517469
0.8	0.4690763	0.5239439	1.1615	58	0.4690763001600726542	0.52394390652487269288	1.1615000475714855626
				59	0.4690763001600726546	0.52394390652487269228	1.1615000475714855631
				60	0.4690763001600726549	0.52394390652487269186	1.1615000475714855635
0.9	0.4588517	0.5116331	1.21232	58	0.4588517197238235193	0.51163307906334812957	1.2123188450384572282
				59	0.4588517197238235193	0.51163307906334812942	1.2123188450384572283
				60	0.4588517197238235194	0.51163307906334812931	1.2123188450384572284
1.0	0.4490278	0.4997484	1.27004	58	0.4490278062878924345	0.49974842186381900536	1.2700424270699528174
				59	0.4490278062878924346	0.49974842186381900534	1.2700424270699528174
				60	0.4490278062878924346	0.49974842186381900532	1.2700424270699528174

† Calculated here (see Eq. (138)) from $\varepsilon_{p,1}$ and $\varepsilon_{p,2}$ as reported by Garcia and Siewert.

Table 2
Values of $\varepsilon_{p,1}$, $\varepsilon_{p,1}$, and $\hat{\mu}$ presented in the dimensionless form used by Garcia and Siewert [19] and computed using the same molecular masses and sizes that they used for a binary, rigid-sphere, gas mixture of He:Xe with the current order 58, 59, and 60 results arbitrarily truncated at 20 significant figures.

	Garcia and Siewert [19]			Present work			
x_1	$\varepsilon_{p,1}$	$\varepsilon_{p,2}$	$\hat{\mu}^{\dagger}$	m	$\varepsilon_{p,1}$	$\varepsilon_{p,2}$	$\hat{\mu}$
0.0	0.6872939	2.571784	1.46801	58	0.6872937322013061374	2.571784078122286320	1.468010550302649127
				59	0.6872937447070151995	2.571784078122286320	1.468010550302649127
				60	0.6872937560689973469	2.571784078122286320	1.468010550302649127
0.1	0.6591318	2.496050	1.47828	58	0.6591317010899825315	2.496050032005066600	1.478278852837708127
				59	0.6591317080786303823	2.496050031666878712	1.478278853089907215
				60	0.6591317143929958823	2.496050031359873829	1.478278853316941398
0.2	0.6313460	2.424505	1.48925	58	0.6313459115366166018	2.424504832197169971	1.489249176338942153
				59	0.6313459151919377899	2.424504831768158172	1.489249176618539411
				60	0.6313459184749826248	2.424504831380797925	1.489249176868483918
0.3	0.6040327	2.358467	1.50078	58	0.6040326461214347863	2.358466807576086552	1.500783073486171713
				59	0.6040326478783327377	2.358466807190135265	1.500783073696612578
				60	0.6040326494459975592	2.358466806843711164	1.500783073883215617
0.4	0.5773241	2.299872	1.51252	58	0.5773240572503967573	2.299872149675410386	1.512521641308333831
				59	0.5773240580043236595	2.299872149389605219	1.512521641430480553
				60	0.5773240586719789073	2.299872149134824764	1.512521641537703647
0.5	0.5514060	2.251684	1.52364	58	0.5514060290073501917	2.251684192827048473	1.523643416565273163
				59	0.5514060292816985444	2.251684192650492136	1.523643416618428841
				60	0.5514060295223141155	2.251684192494498734	1.523643416664425937
0.6	0.5265471	2.218677	1.53232	58	0.5265471195182058579	2.218676851307744181	1.532318201715292502
				59	0.5265471195936374130	2.218676851223210552	1.532318201729866321
				60	0.5265471196587474937	2.218676851149646338	1.532318201742141638
0.7	0.5031460	2.209062	1.53438	58	0.5031459951586207703	2.209061619755904188	1.534376355824601918
				59	0.5031459951685818242	2.209061619735066384	1.534376355825692539
				60	0.5031459951766715873	2.209061619717991686	1.534376355826509563
0.8	0.4818064	2.238244	1.51971	58	0.4818063818842309327	2.238244420339314744	1.519710173141090702
				59	0.4818063818813508385	2.238244420352276109	1.519710173141616425
				60	0.4818063818786078540	2.238244420364739004	1.519710173142160379
0.9	0.4634256	2.338849	1.46101	58	0.4634255855727828224	2.338848604878315598	1.461014354508504662
				59	0.4634255855713369549	2.338848604897930042	1.461014354509986315
				60	0.4634255855700366116	2.338848604915747730	1.461014354511358659
1.0	0.4490278	2.593733	1.27004	58	0.4490278062878924345	2.593733186936566237	1.270042427069952817
				59	0.4490278062878924346	2.593733186936564630	1.270042427069952817
				60	0.4490278062878924346	2.593733186936563393	1.270042427069952817

† Calculated here (see Eq. (138)) from $\varepsilon_{p,1}$ and $\varepsilon_{p,2}$ as reported by Garcia and Siewert.

Table 3

Values of $\varepsilon_{p,1}$, $\varepsilon_{p,1}$, and $\hat{\mu}$ computed in the current work using the same molecular masses and sizes as Garcia and Siewert [19] for a binary, rigid-sphere, gas mixture of Ne:Ar with $x_1 = 0.5$ and demonstrating the degree of convergence of the current results as a function of the order of the approximation up to order $m = 60$.

m	$\varepsilon_{p,1}$	$\varepsilon_{p,2}$	$\hat{\mu}$
1	0.489014271942809503	0.557640318263336575	1.02279153639148582
2	0.500148966484225233	0.562686482783745180	1.03860348223764872
3	0.501287238352212758	0.562824661265745622	1.03985083033778413
4	0.501485684809746423	0.562812826116856584	1.04003318702819818
5	0.501531933091506243	0.562805024536433164	1.04007075716938611
6	0.501544924542130930	0.562801913569808127	1.04008041238468336
7	0.501549089164067012	0.562800713769189732	1.04008330960997527
8	0.501550563518324078	0.562800237499302848	1.04008428493866153
9	0.501551127737287012	0.562800040557032337	1.04008464384168076
10	0.501551357715680362	0.562799955697608877	1.04008478565203115
⋮	⋮	⋮	⋮
15	0.501551536471126114	0.562799885419491511	1.04008489165614598
⋮	⋮	⋮	⋮
20	0.501551541612327209	0.562799883193865289	1.04008489450524761
⋮	⋮	⋮	⋮
25	0.501551541875005921	0.562799883075527972	1.04008489464629812
⋮	⋮	⋮	⋮
30	0.501551541894301922	0.562799883066649675	1.04008489465647830
⋮	⋮	⋮	⋮
35	0.501551541896133184	0.562799883065796074	1.04008489465743368
⋮	⋮	⋮	⋮
40	0.501551541896343999	0.562799883065696938	1.04008489465754281
⋮	⋮	⋮	⋮
45	0.501551541896372260	0.562799883065683563	1.04008489465755736
⋮	⋮	⋮	⋮
50	0.501551541896376550	0.562799883065681524	1.04008489465755955
51	0.501551541896376807	0.562799883065681401	1.04008489465755968
52	0.501551541896376988	0.562799883065681314	1.04008489465755978
53	0.501551541896377116	0.562799883065681253	1.04008489465755984
54	0.501551541896377207	0.562799883065681210	1.04008489465755989
55	0.501551541896377272	0.562799883065681179	1.04008489465755992
56	0.501551541896377318	0.562799883065681157	1.04008489465755995
57	0.501551541896377351	0.562799883065681141	1.04008489465755996
58	0.501551541896377375	0.562799883065681130	1.04008489465755997
59	0.501551541896377392	0.562799883065681121	1.04008489465755998
60	0.501551541896377404	0.562799883065681115	1.04008489465755999

Table 4

Values of the Chapman–Enskog solution for viscosity presented in the dimensionless form used by Garcia and Siewert [19] and computed using the same molecular masses and sizes that they used for a binary, rigid-sphere, gas mixture of Ne:Ar with the current order 60 results arbitrarily truncated at 8 significant figures.

		Garcia and Siewert [19]		Present work (order 60)	
	$\mathscr{C}$	$\widehat{B}_1(\mathscr{C})$	$\widehat{B}_2(\mathscr{C})$	$\widehat{B}_1(\mathscr{C})$	$\widehat{B}_2(\mathscr{C})$
$x_1 = 0.1$	0.0	0.0	0.0	0.0	0.0
	0.1	0.008948039	0.008420111	0.0089479790	0.0084201111
	0.2	0.03545276	0.03353171	0.035452636	0.033531712
	0.3	0.07855316	0.07490027	0.078553089	0.074900268
	0.4	0.1368178	0.1318384	0.13681786	0.13183841
	0.5	0.2085412	0.2034548	0.20854121	0.20345477
	1.0	0.7100345	0.7433387	0.71003446	0.74333869
	1.5	1.339594	1.488148	1.3395936	1.4881476
	2.0	2.019132	2.338906	2.0191322	2.3389055
	2.5	2.717517	3.240245	2.7175174	3.2402453
	3.0	3.421611	4.163716	3.4216110	4.1637163
	3.5	4.125616	5.094889	4.1256150	5.0948888
	4.0	4.826999	6.026471	4.8269989	6.0264714
	4.5	5.524760	6.954864	5.5247734	6.9548639
	5.0	6.218626	7.878411	6.2187147	7.8784110
$x_1 = 0.5$	0.0	0.0	0.0	0.0	0.0
	0.1	0.007690682	0.007288256	0.0076906715	0.0072882559
	0.2	0.03053225	0.02905193	0.030532226	0.029051931
	0.3	0.06786436	0.06499422	0.067864348	0.064994222
	0.4	0.1186811	0.1146411	0.11868112	0.11464110
	0.5	0.1817459	0.1773693	0.18174593	0.17736928
	1.0	0.6346182	0.6592924	0.63461815	0.65929239
	1.5	1.221164	1.345306	1.2211640	1.3453057
	2.0	1.865655	2.151078	1.8656546	2.1510783
	2.5	2.534725	3.023796	2.5347254	3.0237958
	3.0	3.213507	3.933287	3.2135065	3.9332871
	3.5	3.895101	4.862534	3.8951010	4.8625341
	4.0	4.576255	5.801829	4.5762547	5.8018289
	4.5	5.255462	6.745558	5.2554652	6.7455579
	5.0	5.932088	7.690450	5.9321066	7.6904498
$x_1 = 0.9$	0.0	0.0	0.0	0.0	0.0
	0.1	0.006486759	0.006248353	0.0064867580	0.0062483533
	0.2	0.02581069	0.02493178	0.025810685	0.024931775
	0.3	0.05757558	0.05586826	0.057575575	0.055868264
	0.4	0.1011612	0.09876387	0.10116121	0.098763868
	0.5	0.1557752	0.1532257	0.15577522	0.15322573
	1.0	0.5615379	0.5805261	0.56153792	0.58052610
	1.5	1.109914	1.211018	1.1099144	1.2110177
	2.0	1.727389	1.976274	1.7273894	1.9762740
	2.5	2.375998	2.826901	2.3759977	2.8269010
	3.0	3.037333	3.731066	3.0373327	3.7310662
	3.5	3.702490	4.668925	3.7024900	4.6689249
	4.0	4.367176	5.628092	4.3671764	5.6280918
	4.5	5.029391	6.600728	5.0293911	6.6007275
	5.0	5.688295	7.581783	5.6882965	7.5817829

and Ar, respectively, with a molecular diameter ratio of $\sigma_{\mathrm{Ar}}/\sigma_{\mathrm{Ne}} = 1.406$. Additionally, for the He:Xe mixture, Garcia and Siewert have used molecular weights of 4.0026 and 131.30 for He and Xe, respectively, with a molecular diameter ratio of $\sigma_{\mathrm{Xe}}/\sigma_{\mathrm{He}} = 2.226$. For our comparisons with their results in Tables 1–5, we have used the same molecular parameters used by Garcia and Siewert as listed above and note only that the values we report as our primary results were obtained with slightly different parameter values that we specify later. We note from the results presented in Tables 1–3 that we have precision to a high number of digits in our results for $\varepsilon_{p,1}$, $\varepsilon_{p,2}$, and $\hat{\mu}$, and that these results support those of Garcia and Siewert to the number of digits that they had reported. Here, note that our results for $\widehat{B}_1$ and $\widehat{B}_2$ have not necessarily converged fully to the number of digits shown for all values of $\mathscr{C}$ and that, for the lower and higher values of $\mathscr{C}$, expansions beyond order 60 will be needed for more complete convergence. However, we note that it is the middle range of $\mathscr{C}$ (≈ 1–3) that contributes the most to the macroscopic quantities of interest (such as viscosity and the slip coefficients) and, thus, our results are the most appropriate ones to use in calculations of such quantities.

Finally, we present the primary results of this work for binary mixtures of the noble gases He, Ne, Ar, Kr, and Xe. We note that a more transparent way to report viscosity results may be achieved by normalizing the viscosity in the following manner:

$$[\mu]_m = [\mu]_1 [\mu]^\star_m, \tag{144}$$

where, $[\mu]^\star_m \equiv [\mu]_m/[\mu]_1$ and $[\mu]_1$ is the viscosity of the mixture computed with a first-order approximation ($m = 1$). In the general case, $[\mu]_1$ can be explicitly expressed as:

$$\begin{aligned}[\mu]_1 = 5kT\big(&3x_1^2\Omega_1^{(2)}(2) + 3x_2^2\Omega_2^{(2)}(2) + 20M_1M_2\Omega_{12}^{(1)}(1)\\ &+ 6(M_1x_1 - M_2x_2)^2\Omega_{12}^{(2)}(2)\big)/\Delta,\end{aligned} \tag{145}$$

where:

$$\begin{aligned}\Delta = 8\big\{&3M_1^2x_1\Omega_1^{(2)}(2)\big(x_2\Omega_2^{(2)}(2) + 2x_1\Omega_{12}^{(2)}(2)\big)\\ &+ 3M_2^2x_2\Omega_2^{(2)}(2)\big(x_1\Omega_1^{(2)}(2) + 2x_2\Omega_{12}^{(2)}(2)\big)\\ &+ 2M_1M_2\big[10x_1^2\Omega_1^{(2)}(2)\Omega_{12}^{(1)}(1) + 10x_2^2\Omega_2^{(2)}(2)\Omega_{12}^{(1)}(1)\\ &+ x_1x_2\big(3\Omega_1^{(2)}(2)\Omega_2^{(2)}(2) + 20\Omega_{12}^{(1)}(1)\Omega_{12}^{(2)}(2)\big)\big]\big\},\end{aligned} \tag{146}$$

and, for the specific case of rigid-sphere molecules, one would use Eqs. (113)–(115) for the omega integrals needed. Based upon this definition, one may express Eq. (131) as:

Table 5
Values of the Chapman–Enskog solution for viscosity presented in the dimensionless form used by Garcia and Siewert [19] and computed using the same molecular masses and sizes that they used for a binary, rigid-sphere, gas mixture of He:Xe with the current order 60 results arbitrarily truncated at 8 significant figures.

		Garcia and Siewert [19]		Present work (order 60)	
	$\mathscr{C}$	$\widehat{B}_1(\mathscr{C})$	$\widehat{B}_2(\mathscr{C})$	$\widehat{B}_1(\mathscr{C})$	$\widehat{B}_2(\mathscr{C})$
$x_1 = 0.1$	0.0	0.0	0.0	0.0	0.0
	0.1	0.03050749	0.03378962	0.027923925	0.033789624
	0.2	0.1068797	0.1345907	0.10266667	0.13459069
	0.3	0.2053841	0.3007407	0.20394473	0.30074069
	0.4	0.3129096	0.5295988	0.31389900	0.52959878
	0.5	0.4244460	0.8177201	0.42507796	0.81772012
	1.0	1.009060	2.996638	1.0088808	2.9966377
	1.5	1.627307	6.012572	1.6271621	6.0125720
	2.0	2.276993	9.460218	2.2771273	9.4602184
	2.5	2.956657	13.11015	2.9569973	13.110150
	3.0	3.664323	16.84482	3.6639151	16.844819
	3.5	4.397634	20.60532	4.3942343	20.605319
	4.0	5.154014	24.36247	5.1539701	24.362467
	4.5	5.930801	28.10232	6.0289644	28.102317
	5.0	6.725367	31.81886	7.2682967	31.818855
$x_1 = 0.5$	0.0	0.0	0.0	0.0	0.0
	0.1	0.01527023	0.02786601	0.015108419	0.027866010
	0.2	0.05814503	0.1111796	0.057835996	0.11117959
	0.3	0.1219944	0.2490995	0.12186368	0.24909951
	0.4	0.2002270	0.4402630	0.20031454	0.44026295
	0.5	0.2880717	0.6828407	0.28813920	0.68284071
	1.0	0.7973243	2.579754	0.79730316	2.5797542
	1.5	1.359373	5.352794	1.3593535	5.3527938
	2.0	1.950200	8.671471	1.9502172	8.6714712
	2.5	2.564073	12.30070	2.5641182	12.300701
	3.0	3.198823	16.09584	3.1987692	16.095839
	3.5	3.853043	19.97289	3.8525839	19.972889
	4.0	4.525390	23.88402	4.5253686	23.884024
	4.5	5.214438	27.80230	5.2276812	27.802303
	5.0	5.918679	31.71282	5.9926429	31.712822
$x_1 = 0.9$	0.0	0.0	0.0	0.0	0.0
	0.1	0.007397869	0.02506036	0.0074148273	0.025060360
	0.2	0.02936495	0.1001712	0.029393296	0.10017122
	0.3	0.06515651	0.2251231	0.065163889	0.22512312
	0.4	0.1135534	0.3995709	0.11354393	0.39957090
	0.5	0.1731147	0.6230394	0.17311118	0.62303935
	1.0	0.5922252	2.451043	0.59222745	2.4510430
	1.5	1.128489	5.375200	1.1284900	5.3752000
	2.0	1.717866	9.250880	1.7178641	9.2508803
	2.5	2.333036	13.92669	2.3330328	13.926686
	3.0	2.962572	19.26161	2.9625778	19.261605
	3.5	3.601714	25.13209	3.6017485	25.132086
	4.0	4.248524	31.43345	4.2485073	31.433451
	4.5	4.902269	38.07875	4.9012010	38.078753
	5.0	5.562708	44.99667	5.5575101	44.996673

$$[\mu]_m = [\mu]_1 \frac{p}{[\mu]_1} \left(x_1 \frac{b_1^{(m)}}{b_1^{(1)}} b_1^{(1)} + x_2 \frac{b_{-1}^{(m)}}{b_{-1}^{(1)}} b_{-1}^{(1)} \right), \tag{147}$$

such that:

$$[\mu]_m = [\mu]_m^\star [\mu]_1 = p\big(x_1 b_1^{(m)\star} b_1^{(1)} + x_2 b_{-1}^{(m)\star} b_{-1}^{(1)}\big), \tag{148}$$

where:

$$b_1^{(m)\star} \equiv \frac{b_1^{(m)}}{b_1^{(1)}}, \tag{149}$$

$$b_{-1}^{(m)\star} \equiv \frac{b_{-1}^{(m)}}{b_{-1}^{(1)}}, \tag{150}$$

are also normalized to their order 1 values, $b_1^{(1)}$ and $b_{-1}^{(1)}$, which are, explicitly:

$$\begin{aligned} b_1^{(1)} &= n^{-1}\big(5\big(3x_1 \Omega_1^{(2)}(2) + 20 M_1 M_2 \Omega_{12}^{(1)}(1) \\ &\quad + 6\big(M_2^2 x_2 - M_1 M_2 x_1\big)\Omega_{12}^{(2)}(2)\big)\big)/\Delta, \end{aligned} \tag{151}$$

$$\begin{aligned} b_{-1}^{(1)} &= n^{-1}\big(5\big(3x_2 \Omega_2^{(2)}(2) + 20 M_1 M_2 \Omega_{12}^{(1)}(1) \\ &\quad + 6\big(M_1^2 x_1 - M_1 M_2 x_2\big)\Omega_{12}^{(2)}(2)\big)\big)/\Delta. \end{aligned} \tag{152}$$

Again, for the specific case of rigid-sphere molecules, one uses Eqs. (113)–(115) for the omega integrals needed. We have reported some representative order 60 results for $[\mu]_1$, $b_1^{(1)}$, $b_{-1}^{(1)}$, and $[\mu]_{60}^\star$, $b_1^{(60)\star}$, $b_{-1}^{(60)\star}$ for binary mixtures of the noble gases He, Ne, Ar, Kr, and Xe in Tables 6 and 7, respectively. The molecular mass and diameter values used in computing the results given in Tables 6 and 7 are summarized in Table 8.

As a result of our obtaining normalized viscosity values for each order of approximation up to 60, we have for each combination of parameters considered, a sequence of values that is known to be increasing monotonically to a fixed limiting value of the normalized viscosity. As a part of our work, we have applied the *Mathematica*® function `SequenceLimit` to each such sequence of values in order to determine the limiting values associated with our order 60 results. The `SequenceLimit` function extrapolates to the limit using the Wynn-epsilon algorithm. The extrapolated results, which we have included in Table 9, are truncated at the number of digits that appear to be common to all of the extrapolations done for each point using Wynn orders from 12 to 22 which is the range of Wynn orders that appears to give the most consistent extrapolated values. It is our assessment that the extrapolated values presented in Table 9 are likely correct to the

Table 6
Values of $[\mu]_1$, $b_1^{(1)}$, and $b_{-1}^{(1)}$ for binary, rigid-sphere gas mixtures of He, Ne, Ar, Kr, and Xe (arbitrarily truncated at 21 significant figures).

	x_1	$[\mu]_1 \times 10^4$ $(\mathrm{gm\,cm^{-1}\,s^{-1}})$	$b_1^{(1)} \times 10^{10}$ $(\mathrm{s^{-1}})$	$b_{-1}^{(1)} \times 10^{10}$ $(\mathrm{s^{-1}})$
He:Ne	0.0	2.92741878308280513703	2.88913770844589700176	1.34368924951280000719
	0.1	2.87457091754218394474	2.99860390470612425053	1.38237406112066703749
	0.2	2.81409265010785212121	3.11589653970997464552	1.42288139658991696790
	0.3	2.74472292566742726819	3.24178717070666655686	1.46526632178454263040
	0.4	2.66495469170310718033	3.37713346770320234228	1.50956427340968496508
	0.5	2.57297981381332066176	3.52288635057964958548	1.55578093550635221419
	0.6	2.46662044687919315972	3.68009563162441021352	1.60387809422126770323
	0.7	2.34324347097656398304	3.84991258242714015516	1.65375389479859582585
	0.8	2.19965389952399640659	4.03358687577877131889	1.70521529625885566101
	0.9	2.03196249535873792164	4.23245384538498686289	1.75793964358374222415
	1.0	1.83542239893324326724	4.44790567000913691538	1.81142106975893734739
He:Ar	0.0	2.08288258264550698739	2.05564528265038932879	0.829153625748764596205
	0.1	2.08912595200792372568	2.19336527037006398775	0.877782658038027373833
	0.2	2.09429607819828807954	2.35054849330352297306	0.932353661615851752165
	0.3	2.09772294109462499412	2.53156135931888861716	0.993995420556691406144
	0.4	2.09838608792901871679	2.74215695429361885582	1.06412970038542593978
	0.5	2.09470554052516077882	2.99006118786990180082	1.14456608185654434537
	0.6	2.08418333820625434606	3.28587620906805406215	1.23763090825063251035

(continued on next page)

Table 6 (*continued*)

	x_1	$[\mu]_1 \times 10^4$ $(\mathrm{gm\,cm^{-1}\,s^{-1}})$	$b_1^{(1)} \times 10^{10}$ $(\mathrm{s^{-1}})$	$b_{-1}^{(1)} \times 10^{10}$ $(\mathrm{s^{-1}})$
	0.7	2.06276663965863243481	3.64451499737534967737	1.34633984467443589106
	0.8	2.02366815070522618899	4.08755640237272916364	1.47461737696076070069
	0.9	1.95507315698396541303	4.64725512274322074671	1.62753519387231249376
	1.0	1.83542239893324326724	5.37364030429427803665	1.81142106975893734739
He:Kr	0.0	2.29069445315765001595	2.26073965275859858470	0.665778798598662287728
	0.1	2.29996588414412040408	2.44303615068828806806	0.711573080954387442542
	0.2	2.30864526678849505724	2.65705292525240597026	0.764066940927188639269
	0.3	2.31598174702150130554	2.91178412864209581937	0.824824604585504706833
	0.4	2.32068645519376118207	3.21994761558125931037	0.895927220086687846857
	0.5	2.32050109284246933754	3.60011551554361606980	0.980197522388719163445
	0.6	2.31134391844667025898	4.08048097387730268622	1.08154450157776047415
	0.7	2.28551296355833244016	4.70592443991018336117	1.20549798306247147150
	0.8	2.22760711632037560795	5.55227656735689917586	1.36002747340920304707
	0.9	2.10432159509426816375	6.75790474954513806567	1.55668163099663574224
	1.0	1.83542239893324326724	8.60270648697165432120	1.81142106975893734739
He:Xe	0.0	2.07245362110288412252	2.04535269785628830251	5.21793887055332997073
	0.1	2.08699110979597286944	2.22608453023373598385	5.62240064767561632089
	0.2	2.10275645301134304187	2.44171000558585276358	6.09456316227242988984
	0.3	2.11966104887099464023	2.70336656086767784013	6.65287380250258870534
	0.4	2.13735634836514919031	3.02747173775668941418	7.32309167026819296708
	0.5	2.15491994307889401534	3.43925548062224684473	8.14225778847566163596
	0.6	2.17012784001680663964	3.97954826746530917399	9.16550583457713350475
	0.7	2.17763803894805052416	4.71893608547599563561	10.4782974497266187420
	0.8	2.16397106269447652236	5.79057815385998744863	12.2194719937669635565
	0.9	2.09150692441684148171	7.47807843578246574375	14.6261000291820395342
	1.0	1.83542239893324326724	10.5055599727433085104	18.1142106975893734739
Ne:Ar	0.0	2.08288258264550698739	2.05564528265038932879	1.79079008360150007741
	0.1	2.13444789659795181421	2.13381497011060281924	1.86102817646030435012
	0.2	2.19070714612184236145	2.21830122856319785949	1.93709435109062016556
	0.3	2.25232215799943131862	2.30991769133478242145	2.01975585978436883506
	0.4	2.32008396966820725237	2.40962647579006482559	2.10992241231590757870
	0.5	2.39494573365483538915	2.51857417157399298047	2.20868116255842328234
	0.6	2.47806615998287277431	2.63813895189007635444	2.31734261498778654141
	0.7	2.57086763987752163732	2.76999307293261056454	2.43750166794660934188
	0.8	2.67511516124266597363	2.91618701681492024662	2.57111998801520156137
	0.9	2.79302520834962376628	3.07926463832057530512	2.72063899868058390053
	1.0	2.92741878308280513703	3.26242363787507244099	2.88913770844589700176
Ne:Kr	0.0	2.29069445315765001595	2.26073965275859858470	1.43875513127484465816
	0.1	2.32626768312174477996	2.38260190450511323634	1.51505987027801028291
	0.2	2.36553152923020777062	2.51827892104716194002	1.59987483754936245766
	0.3	2.40908648566156674660	2.67024752790335195916	1.69470078232722028380
	0.4	2.45766924144659698770	2.84161064022732404186	1.80141143034886586838
	0.5	2.51219299398853531394	3.03630711693035922887	1.92237631556613287858
	0.6	2.57380265744499891992	3.25941146431176393440	2.06063523473877950046
	0.7	2.64395158679786193943	3.51757134460689872021	2.22015133385637432725
	0.8	2.72450983718089370522	3.81966222856419355921	2.40618712874705141925
	0.9	2.81791925712920618360	4.17779481300207183895	2.62587844143078789869
	1.0	2.92741878308280513703	4.60891576019373779941	2.88913770844589700176
Ne:Xe	0.0	2.07245362110288412252	2.04535269785628830251	1.12659967856692117720 9
	0.1	2.11029336364559657894	2.18077584471551245090	1.19999359934273759839 6
	0.2	2.15315130597330988644	2.33534557640444017216	1.28359329750738062123 0
	0.3	2.20209227720914312297	2.51341698597652313610	1.37968071216595008015 2
	0.4	2.25850364545503902581	2.72076918202165363878	1.49127071612379643748 1
	0.5	2.32422656317606976570	2.96523656275628933770	1.62242998187942695285 0
	0.6	2.40175634604303764907	3.25770687687340477821	1.77877744690557890035 1
	0.7	2.49455796191427376289	3.61375987934964924373	1.96829904139997058126 9
	0.8	2.60758047145323730071	4.05648018816788498053	2.20273224934741599976 1
	0.9	2.74812998659531872419	4.62154986037212065587	2.50004270854852493213 3
	1.0	2.92741878308280513703	5.36708604328363454937	2.88913770844589700176 2
Ar:Kr	0.0	2.29069445315765001595	2.26073965275859858470	1.66101342369596463614 3
	0.1	2.27792580456086075762	2.30967533841138706717	1.69430171803658136088 4
	0.2	2.26383490878451817545	2.36058777623194628792	1.72880561248903288659 3
	0.3	2.24828394295070345281	2.41358491132955562281	1.76458098517125713772 9
	0.4	2.23111866690908974380	2.46878107410759718559	1.80168569678109063613 3
	0.5	2.21216619817109373792	2.52629719333286078498	1.84017938830265609224 4
	0.6	2.19123245210886234367	2.58626095040217083954	1.88012318268091537529 3
	0.7	2.16809919349290761120	2.64880684858500104682	1.92157926290124693242 6
	0.8	2.14252063667854862465	2.71407616272158469824	1.96461029151618490692 1
	0.9	2.11421952175011211605	2.78221672408010974717	2.00927862728041779304 5
	1.0	2.08288258264550698739	2.85338248109014473994	2.05564528265038932879 1
Ar:Xe	0.0	2.07245362110288412252	2.04535269785628830251	1.31599130047056829750
	0.1	2.07885303892681738921	2.12786532131841598307	1.36589643013590815535
	0.2	2.08487334166880826240	2.21711018318303104219	1.41960931251242684238

Table 6 (*continued*)

	x_1	$[\mu]_1 \times 10^4$ $(\mathrm{gm\,cm^{-1}\,s^{-1}})$	$b_1^{(1)} \times 10^{10}$ $(\mathrm{s^{-1}})$	$b_{-1}^{(1)} \times 10^{10}$ $(\mathrm{s^{-1}})$
	0.3	2.09034736388545465690	2.31391855913599781331	1.47756485847280140756
	0.4	2.09505717958736839229	2.41925908328860085569	1.54026306415510363204
	0.5	2.09871763017990481586	2.53426592941643817744	1.60828058956684297897
	0.6	2.10095380481519096206	2.66027374032161325255	1.68228449356499894861
	0.7	2.10126994229715787569	2.79886105163642599614	1.76304840030450147477
	0.8	2.09900602272995144993	2.95190436678978995862	1.85147125931408896474
	0.9	2.09327647662780293721	3.12164546382420226068	1.94859856896764058793
	1.0	2.08288258264550698739	3.31077483494224526969	2.05564528265038932879
Kr:Xe	0.0	2.07245362110288412252	2.04535269785628830251	1.79353101667798172505
	0.1	2.09014777588804023440	2.08853033226441748996	1.83138171860657601645
	0.2	2.10860200168842971318	2.13356950439597467224	1.87086386096744457862
	0.3	2.12786634714630409637	2.18059320630540174178	1.91208520682306201486
	0.4	2.14799535353944957717	2.22973550492645557374	1.95516322062524281055
	0.5	2.16904857045027546492	2.28114281691041605543	2.00022618471855106012
	0.6	2.19109114412954451343	2.33497536365499571956	2.04741447362476313561
	0.7	2.21419449079810592301	2.39140883694560743056	2.09688201274197582646
	0.8	2.23843706954468899106	2.45063631164389853508	2.14879795335560443243
	0.9	2.26390527244181719856	2.51287044924241225284	2.20334860232182446697
	1.0	2.29069445315765001595	2.57834604521498857140	2.26073965275859858470

Table 7
Values of $[\mu]_{60}^{\star}$, $b_1^{(60)\star}$, and $b_{-1}^{(60)\star}$ for binary, rigid-sphere gas mixtures of He, Ne, Ar, Kr, and Xe (arbitrarily truncated at 21 significant figures).

	x_1	$[\mu]_{60}^{\star}$	$b_1^{(60)\star}$	$b_{-1}^{(60)\star}$
He:Ne	0.0	1.01603394165596225394	1.01603394165596225394	1.04485118491390073388
	0.1	1.01507981005595704974	1.01371253815614662426	1.04177241284408011604
	0.2	1.01425878361375016253	1.01146554561865102285	1.03872587025744242162
	0.3	1.01358127106503753285	1.00929376018953282452	1.03571476375207361499
	0.4	1.01306346604592649143	1.00719893038545592346	1.03274330346693874360
	0.5	1.01272954522889912551	1.00518354872922256718	1.02981658329674914769
	0.6	1.01261532435082903314	1.00325053005258540094	1.02694032769208383891
	0.7	1.01277457235078867075	1.00140267898088736701	1.02412039477035941321
	0.8	1.01329052801883149671	0.999641789518708382692	1.02136185083153100733
	0.9	1.01429844951434984249	0.997967117200199526458	1.01866730196333090790
	1.0	1.01603394165596225394	0.996372787771616987210	1.01603394165596225394
He:Ar	0.0	1.01603394165596225394	1.01603394165596225394	1.05792170948470323283
	0.1	1.01587330369910437037	1.01414040770254626373	1.05484407254589890505
	0.2	1.01572351651591546094	1.01216930500376273738	1.05156548017718878262
	0.3	1.01558510378211889495	1.01011928485306677439	1.04806660553474875775
	0.4	1.01545990027647383972	1.00799171156045055591	1.04432707541903814538
	0.5	1.01535234856768505558	1.00579255958604055537	1.04032631420527959139
	0.6	1.01527198434076274506	1.00353557181296176032	1.03604522050124807820
	0.7	1.01523823494731039261	1.00124756995304609840	1.03146928492961485769
	0.8	1.01528982741619837232	0.998977447114818741648	1.02659407751873486611
	0.9	1.01550365889632625106	0.996811310231086621115	1.02143410673055207608
	1.0	1.01603394165596225394	0.994896796246408654503	1.01603394165596225394
He:Kr	0.0	1.01603394165596225394	1.01603394165596225394	1.06672812265169489641
	0.1	1.01613267018074011140	1.01459883217027165082	1.06352765501439215433
	0.2	1.01619235268418533591	1.01304035552809960009	1.06003680041700754320
	0.3	1.01620154994363010615	1.01134395742205251696	1.05621402657755079311
	0.4	1.01614732639360126813	1.00949475081562635451	1.05201119634227042454
	0.5	1.01601694909473345520	1.00747955862365593122	1.04737347795258903733
	0.6	1.01580289424595185290	1.00529171115686511912	1.04224081553684045984
	0.7	1.01551637720177399937	1.00294218768787700613	1.03655327407293334150
	0.8	1.01522390859716212394	1.00048608938610277721	1.03026559890478261044
	0.9	1.01515202001727141556	0.998088657774696332632	1.02338267442456988958
	1.0	1.01603394165596225394	0.996201924433832422135	1.01603394165596225394
He:Xe	0.0	1.01603394165596225394	1.01603394165596225394	1.07158326797078869468
	0.1	1.01639357708144408583	1.01492473740289806586	1.06873394450867289307
	0.2	1.01672843374133128104	1.01368207158062152843	1.06554789968428277036
	0.3	1.01702098365247956234	1.01228122347753187156	1.06196054250931617906
	0.4	1.01724632590091958839	1.01069209533799732781	1.05789052786936544082
	0.5	1.01736974570885297972	1.00887886633944532755	1.05323486342579607405
	0.6	1.01734493704325970319	1.00680164772579403206	1.04786336813622424981
	0.7	1.01711728601114110855	1.00442491574089949417	1.04161464803795254707
	0.8	1.01664931897814380161	1.00174876710053285946	1.03430204780042093843
	0.9	1.01603761248639342751	0.998923328835579890353	1.02576010669253960408
	1.0	1.01603394165596225394	0.996718146433463758077	1.01603394165596225394
Ne:Ar	0.0	1.01603394165596225394	1.01603394165596225394	1.03305997324087952532
	0.1	1.01625590361214093518	1.01475812317239841618	1.03171180980538410381
	0.2	1.01646451044346802436	1.01344380666026345691	1.03030138092865734756
	0.3	1.01665067349896640643	1.01208909970146827722	1.02882343516367989725
	0.4	1.01680332585790530321	1.01069215442363076501	1.02727217561287563806

(*continued on next page*)

Table 7 (*continued*)

	x_1	$[\mu]^\star_{60}$	$b_1^{(60)\star}$	$b_{-1}^{(60)\star}$
	0.5	1.01690903414510012054	1.00925127326839768276	1.02564123102489417856
	0.6	1.01695154099883760832	1.00776508072750981716	1.02392365308246182339
	0.7	1.01691123305354701216	1.00623280108999558652	1.02211196489147566422
	0.8	1.01676453670408282857	1.00465470888707492995	1.02019830514161537200
	0.9	1.01648326079756137771	1.00303286546716734436	1.01817474768446145741
	1.0	1.01603394165596225394	1.00137233943072385176	1.01603394165596225394
Ne:Kr	0.0	1.01603394165596225394	1.01603394165596225394	1.04675335943346634153
	0.1	1.01612540646618870920	1.01411947329532110301	1.04451640446092753344
	0.2	1.01626026630626637042	1.01214949805450020490	1.04214244357355706995
	0.3	1.01642925411275517273	1.01012233828276131435	1.03961666789409590470
	0.4	1.01661819150005820727	1.00803714143863930129	1.03692227313408755420
	0.5	1.01680600049140600516	1.00589446989086513147	1.03404027338103467439
	0.6	1.01696189406361729486	1.00369728275624862132	1.03094943390441066323
	0.7	1.01704138561416541021	1.00145260915367030770	1.02762650060263779045
	0.8	1.01698060209827332320	0.999174486510464381038	1.02404710851579557615
	0.9	1.01668819072730193264	0.996889317300041866186	1.02018821018333592048
	1.0	1.01603394165596225394	0.994646100832690172302	1.01603394165596225394
Ne:Xe	0.0	1.01603394165596225394	1.01603394165596225394	1.05449688035856223423
	0.1	1.01635395674827701356	1.01416482405356853467	1.05215920549044405983
	0.2	1.01671720527082156164	1.01219752714433310872	1.04960927580426410750
	0.3	1.01711281297026403361	1.01012523218887672084	1.04681507874419394814
	0.4	1.01752148286086606687	1.00794185078225281187	1.04373801831561978000
	0.5	1.01791071672093858455	1.00564324201284225906	1.04033138506620501858
	0.6	1.01822722536348454433	1.00322957901003245895	1.03653865446611572123
	0.7	1.01838472081151503329	1.00071006367515523901	1.03229200801676070796
	0.8	1.01824406630492819328	0.998112714176179508972	1.02751237740986319173
	0.9	1.01758089599427684252	0.995505933678772390245	1.02211506471325713734
	1.0	1.01603394165596225394	0.993050162341220604997	1.01603394165596225394
Ar:Kr	0.0	1.01603394165596225394	1.01603394165596225394	1.03194763456257257356
	0.1	1.01575993966166860749	1.01457149563101548533	1.03034074221598809712
	0.2	1.01554335978031314600	1.01312738157502405840	1.02873889449595346821
	0.3	1.01538475013551864171	1.01170099549278364452	1.02714153222377502377
	0.4	1.01528498982394893176	1.01029172830238013721	1.02554810305552652977
	0.5	1.01524535096051309078	1.00889894121112361566	1.02395804345100521118
	0.6	1.01526758058658559367	1.00752193392693871623	1.02237075470339823785
	0.7	1.01535400909566456429	1.00615990305337883670	1.02078557071084214731
	0.8	1.01550769463157585123	1.00481188706908076713	1.01920171431473931166
	0.9	1.01573261708649119216	1.00347669296599229713	1.01761823782586966815
	1.0	1.01603394165596225394	1.00215279775046502261	1.01603394165596225394
Ar:Xe	0.0	1.01603394165596225394	1.01603394165596225394	1.04116338481901273030
	0.1	1.01577516863219155504	1.01412351474229087974	1.03893246856972455155
	0.2	1.01558136721294471638	1.01220942237811891995	1.03664624235719889835
	0.3	1.01545117827261505159	1.01029242064257873278	1.03430170219974377261
	0.4	1.01538268746018511181	1.00837374654928482157	1.03189588350851394077
	0.5	1.01537325187006465361	1.00645522811557972980	1.02942592645019163479
	0.6	1.01541927984660133819	1.00453941823365632690	1.02688916311464699540
	0.7	1.01551594556238502883	1.00262975108483850103	1.02428322804411744723
	0.8	1.01565680932678688980	1.00073071149497989684	1.02160618784767811549
	0.9	1.01583329525527487708	0.998847988255290330590	1.01885667075489835608
	1.0	1.01603394165596225394	0.996988538896871317706	1.01603394165596225394
Kr:Xe	0.0	1.01603394165596225394	1.01603394165596225394	1.02674835022770721873
	0.1	1.01604055057380493445	1.01509458068476239591	1.02574971228455756949
	0.2	1.01605093185160637375	1.01414694340762548643	1.02473631354196476978
	0.3	1.01606350866511655414	1.01319085701439224554	1.02370762280798898498
	0.4	1.01607650988788149051	1.01222616907110427076	1.02266310189309286303
	0.5	1.01608794888524984262	1.01125275444132994504	1.02160220980152967364
	0.6	1.01609560012224982802	1.01027052402155875100	1.02052440837830332776
	0.7	1.01609697341268705910	1.00927943577848343835	1.01942916983825879589
	0.8	1.01608928564915532847	1.00827950871904866433	1.01831598673346045032
	0.9	1.01606942988088685058	1.00727084061375568190	1.01718438508745525258
	1.0	1.01603394165596225394	1.00625363054678644209	1.01603394165596225394

precision shown excepting, perhaps, only the last digit and should be considered the best rigid-sphere gas mixture benchmark values available. Even with the last digit considered to be unreliable, the extrapolated values of Table 9 are more than double the precision of order 60 results and are, effectively, equivalent to about order 150 results. We have determined this approximate equivalency by examination of the results for simple gases where error free results to order 200 have been previously generated [17].

8. Discussion and conclusions

Our purpose in this series of papers has been to explore the use of Sonine polynomial expansions to obtain error free results for the transport coefficients and related Chapman–Enskog functions for simple gases and gas mixtures. In our first paper [17], we explored the case of simple gases, and now we have extended our initial work to include an exploration of viscosity and the related Chapman–Enskog viscosity solutions for selected real-gas mixtures.

Table 8
Parameters at STP[†] of the gases considered in this work.

Gas	Molecular weight (amu) [23]	Molecular diameter $\times 10^8$ (cm) [1]
He	4.002602	2.193
Ne	20.1797	2.602
Ar	39.948	3.659
Kr	83.798	4.199
Xe	131.293	4.939

[†] STP signifies a temperature of 0 °C and a pressure of 1 atm.

For specific results, we have focused in this work on rigid-sphere molecules, as the omega integrals are readily available in a simple analytical form. However, we must emphasize that the expressions that we have obtained are completely general and results for any potential model can now readily be obtained subject only to omega integral values for the given potential model being available to sufficient precision.

We have considered Sonine polynomial expansions up to order 60 but the computational tools and programs that we have constructed apply to expansions of arbitrary order and the only limiting factors that we have encountered involve the speed and memory capacity of the computational resources available to us. Nevertheless, because of the high precision of our computations, we have witnessed excellent convergence in all of our results and, via extrapolation, we have been able to obtain viscosity values to a precision of more than 15 digits for all of the rigid-sphere gas mixtures that we have considered.

Since even in a binary-gas mixture the parameter space is very large (involving mole fractions, mass ratios, size ratios, temperature, etc.), it is always useful to have techniques or expressions that make rapid and precise computations possible. It is because we have emphasized the development of a very general set of such computational tools and techniques that our work, particularly our comparisons with the results of Takata et al. [18] and Garcia and Siewert [19] have been relatively straightforward and quick to obtain. It is this same generality that has allowed us to extend our work in a straightforward manner to additional real gas mixtures and which will allow us to complete the much more substantial

Table 9
Values of the normalized viscosity, $[\mu]^{\star}_{60,\mathrm{Wynn}}$, for binary mixtures of the gases He, Ne, Ar, Kr, and Xe obtained by extrapolating the sequence of values, $[\mu]^{\star}_{m}$; $m = 1, 2, 3, \ldots, 60$, to its limit using the *Mathematica*® function `SequenceLimit` that employs the Wynn-epsilon extrapolation algorithm.

x_1	He:Ne	He:Ar	He:Kr
0.0	1.016033941655962253954579863833629348	1.016033941655962253954579863833629348	1.016033941655962253954579863833629348
0.1	1.015079810056074832134717	1.01587330372317453702	1.016132670599328973
0.2	1.014258783613824913206028	1.01572351653535980306	1.0161923530441204303
0.3	1.0135812710650702617819123	1.015585103792673782204	1.0162015501473002976
0.4	1.01306346604593788423444147	1.0154599002808454998784	1.01614732647703301738
0.5	1.012729545228902311582747161	1.015352348569032867462623	1.016016949116604072729
0.6	1.0126153243508297159266358546	1.015271984341028669894611	1.015802894247880386
0.7	1.0127745723507887702381118664	1.01523823494732865650152	1.0155163772025665
0.8	1.01329052801883150411659927535	1.015289827416199673158	1.01522390860002452
0.9	1.01429844951434984261706022	1.01550365889632978464227	1.01515202001928212
1.0	1.016033941655962253954579863833629348	1.016033941655962253954579863833629348	1.016033941655962253954579863833629348
x_1	He:Xe	Ne:Ar	Ne:Kr
0.0	1.016033941655962253954579863833629348	1.016033941655962253954579863833629348	1.016033941655962253954579863833629348
0.1	1.01639357873148544	1.016255903612141000200899434832	1.01612540646633012657713000
0.2	1.01672843543249824	1.01646451044346809410691073383	1.01626026630639998764083589
0.3	1.017020984822171505	1.0166506734989664588174651800740	1.01642925411284162301126604
0.4	1.017246326517223295	1.0168033258579053353927089480029	1.01661819150010258481306746
0.5	1.0173697459456648910	1.0169090341451001372788428611822	1.016806000491424450961145714
0.6	1.017344937096175	1.01695154099883761565841905350941	1.016961894063623336928447243
0.7	1.017117286013916	1.016911233053547014797343725516189	1.0170413856141668592268535558
0.8	1.016649318985848	1.016764536704082829300794095403725	1.016980602098273539624977052011
0.9	1.0160376124971495	1.0164832607975613778534729187741107	1.016688190727301945675343058029
1.0	1.016033941655962253954579863833629348	1.016033941655962253954579863833629348	1.016033941655962253954579863833629348
x_1	Ne:Xe	Ar:Kr	Ar:Xe
0.0	1.016033941655962253954579863833629348	1.016033941655962253954579863833629348	1.016033941655962253954579863833629348
0.1	1.01635395675300562285571	1.0157599396616686610769442568237	1.015775168632201821548823047
0.2	1.016717205275602430599	1.015543359780313196546727130606	1.015581367212953590718189382
0.3	1.017112812973569150954283	1.0153847501355186755488035433106	1.0154511782726203583166875551
0.4	1.0175214828626720532952501	1.01528498982394895063080996630058	1.0153826874601876660550510798
0.5	1.017910716721731470814721	1.01524535096051309989459744693215	1.01537325187006567002236238251
0.6	1.01822722536375476204914565	1.015267580586585597497151761367399	1.01541927984660166710266410454
0.7	1.0183847208115804505447794	1.015354009095664565674068135802869	1.015515945562385110830623044495
0.8	1.018244066304937407785749851	1.0155076946315758516375118924852254	1.01565680932678690379245752937143
0.9	1.01758089599427726964048704629	1.0157326170864911922593341412913891	1.015833295255274878299014942797563
1.0	1.016033941655962253954579863833629348	1.016033941655962253954579863833629348	1.016033941655962253954579863833629348
x_1	Kr:Xe		
0.0	1.016033941655962253954579863833629348		
0.1	1.016040550573804936133606116111538		
0.2	1.01605093185160637569032285914610		
0.3	1.016063508665116555756778958285538		
0.4	1.016076509887881491652620799721914		
0.5	1.0160879488852498433390948212038049		
0.6	1.0160956001222498284307998029109126		
0.7	1.016096973412687059316969605151087		
0.8	1.01608928564915532856774043863274236		
0.9	1.016069429880886850624641174578726821		
1.0	1.016033941655962253954579863833629348		

parametric study which we expect to present as an additional paper in the near future as well as a study using realistic potential models. The attractiveness of our work is also obvious with respect to ternary and higher multiple gas mixtures as no new basic expressions need to be computed and the computations would be straightforward. Finally, we also note that we are in the process of extending our work to studies of thermal conductivity, diffusion, and thermal diffusion and expect to report these results in future papers as well.

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